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Sociological Cycles: The Role of Expectation-Reality Gaps

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Sociological Cycles: The Role of Expectation-Reality Gaps

Abstract

The contribution deals with periodic phenomena that are often met in sociology. After shortly reviewing such phenomena in various disciplines ranging from physics to economy, it is shown that many of the sociological processes can be simply understood in terms of expectation-reality gaps, with memory effects being a necessary prerequisite. A perceived gap between expectation and reality (or in more general but less precise terms between "seeming" and "being") tends to drive a sociological action in the direction of reducing this gap, but – owing to the intrinsic inertia – only after such difference between what is to be expected (what has been promised) and what has been actually achieved has been perceived over quite a time. This constellation causes a periodic behavior given basic (linear) relations between the involved functionalities. As exemplified for the master-example of a hyped technology, a very simple description of typical over-motivation-frustration cycles is thus arrived at. Computer simulations using an alternative approach based on collision events, support the idea that it is memory that leads to the characteristic time behavior. Though exemplified using the technology-example, the general approach can be used as a platform for various other problems, even for oscillations in the field of fashion. In order to understand the amplitude of such oscillations, a model of the respective prehistory is necessary. In this context such diverse topics as innovation jumps or tabooization are addressed. It is also indicated how learning effects can be implemented in a simple manner. All the models are purposefully kept very simple but believed to hit the basic points and hence to enable sharper definitions of otherwise extremely loosely defined terms. Even abstractions such as sociological equivalent circuits are shown to be helpful.

Zusammenfassung

Der Beitrag befasst sich mit periodischen Vorgängen, wie sie häufig in der Soziologie angetroffen werden, und schlägt eine einfache phänomenologische Erklärung hierfür vor. Nach einer kurzen reviewartigen Darstellung zyklischer Phänomene in Disziplinen wie Physik, Chemie, Biologie und Wirtschaftswissenschaften wird gezeigt, dass eine Vielzahl periodischer Vorgänge in der Soziologie auf eine Diskrepanz zwischen Erwartung und Realität (oder in allgemeineren, aber weniger präzisen Termini zwischen „Sein" und „Schein") zurückgeführt werden können, dass diese aber erst zu Schwingungen führt, wenn sie über eine geraume Zeit verspürt wird. Die Tatsache, dass das Zeitintegral eine maßgebliche Rolle spielt, verleiht dem System die nötige Trägheit, die Oszillationen bedingt. Bezogen auf das Musterbeispiel einer übertrieben angepriesenen, neuen Technologie gelingt auf diese Weise eine simple Beschreibung von typischen Begeisterungs-Frustrations-Zyklen, vorausgesetzt dass einfache (lineare) Beziehungen zwischen den entscheidenden Funktionalitäten (erwarteter Erfolg, realer Erfolg und soziologische Aktivität) herrschen. Die Idee wird durch Computersimulationen gestützt, die eine alternative, auf Kontaktereignissen basierende Methode benützen. Allerdings kann das simple Modell auch als Plattform zur Beschreibung verwandter Phänomene, wie etwa Oszillationen im Bereich der Mode dienen. Um die Amplitude solcher Oszillationen zu verstehen, ist ein Modell der jeweiligen Anfangsphase von Nöten: In diesem Kontext werden so diverse Themen wie Innovationssprünge oder Tabuisierung angesprochen. Es wird auch gezeigt, wie auf einfache Weise Lerneffekte berücksichtigt werden können. Alle Modelle sind absichtlich simpel gehalten, spiegeln aber doch halbquantitativ die grundlegenden Punkte wider und ermöglichen so eine Schärfung relevanter Begrifflichkeiten. In diesem Zusammenhang können sogar Abstraktionen wie soziologische Ersatzschaltkreise hilfreich sein.

1. Introduction and Outline

Oscillations are observed in all branches of science and culture, ranging from the behavior of elementary particles, atoms, molecules in simple chemical or physical systems or even in complex organisms, up to oscillations of the behavior of complex organisms such as human beings (FEISTEL and EBELING 1989). Examples of the latter are phenomena as different as waves of public taste, particularly obvious in fashion, or periodically repeated surplus or deficiency of qualified jobs in a certain profession. In the focus of the presentation are those cycles that are characterized by periodic sequences of under-estimation ("hype cycles"). They are not only ubiquitous but also of great impact on and hence of great interest for society. Many of the other oscillation modes, though, can be mapped on this.

If ensembles of simple atomic or molecular particles are considered, there are essentially two approaches to treat such phenomena, (i) thermodynamic considerations that involve relations between reaction rates and driving forces, and (ii) kinetic considerations that are concerned with reaction rates in terms of collision events (encounter events, usually treated by master-equations). These approaches generally assume so-called "mean-field" concepts (a better term would be: homogenizing concepts) in which the averaged overall property of an ensemble of constituents is replaced by the property of a virtual uniform ensemble of representative (average) constituents.

Needless to say that such simple relations could never describe human behavior in most of its respects. The proof is simple: Through reflection a human being is always able by a purposefully imposed feedback loop to counteract a prognosticated behavior. But in particular, if phenomena of large groups are concerned, there can be cases in which a highly individual behavior may not be very relevant as then individual reactions or opinions can be subordinated to an overall behavior (LE BON 1895, NOELLE-NEUMANN 1993, COLEMAN 1904, HELBING et al. 2000, RADOSAVLJEVIC et al. 2009), and the collective behavior may be approached by mean-field concepts. Consider as example growth models of populations (MONTROLL and BADGER 1974, VOLTERRA 1962, KOTOMIN and KUZOVKOV 1996, WEIDLICH and HAAG 1983), models of behavior of crowds in the case of a panic (HELBING et al. 2000) or the observation of behavioral phase transitions (ZHANG et al. 2024). Further simple examples for the usefulness of kinetic equations are mass action problems such as the "restaurant problem" that considers the occupation of seats in a restaurant (MAIER 2023) or prey-predator models treated by reaction- or diffusion-controlled Lotka-Volterra kinetics (MONTROLL and BADGER 1974). The latter has become a standard model to describe oscillatory behavior in non-equilibrium systems. There it is non-linearity connected with a sufficiently complex mechanism that includes autocatalytic elements, which causes oscillations. The reader should note that the applicability of mean-field concepts that are useful for the description of particles to the social context does not at all mean that the behavior of a social individual is identified with that of a particle. To be specific to this paper, it suffices to make the assumptions that (i) the social group under concern that shows a collective (crowd) behavior reacts to common driving forces, and that (ii) the rate of reaction is proportional to the number of relevant events. The first point refers to the applicability of concepts borrowed from thermodynamics, the second to the applicability of chemical-kinetic concepts. Such assumptions are inherent to the field of "social physics" (WEIDLICH and HAAG 1983, MONTROLL and BADGER 1974) which goes back to Thomas HOBBES but became a serious subject only after the establishment of the physics of complex systems (HAKEN 2004, PRIGOGINE 1980). Again, it should be stressed that describability in terms of concepts known in "hard science" does not mean that human beings are seen as "mechanical objects".

The fact that phenomena such as polarization, segregation in bubbles and not least wave-like phenomena occur, is an empirical fact. In spite of the complexity of individual decisions, common crowd phenomena obviously can occur owing to common (not necessarily identical) behavioral properties of individuals such as inclination to adjust to majorities or conversely the wish of being different (see also the theme of fashion addressed below). The quantity of the rate of such phenomena depends definitely on the degree of communication and can be particularly accelerated by today's internet.

The outline of the paper is as follows: First we start with a brief review of periodic processes and their description in various disciplines. Then, emphasis is laid on collective, periodic phenomena in sociology which are characterized by memory effects. (To avoid misunderstandings, it cannot be emphasized enough that in this context memory is meant in the sense of system theory i.e. the output of a process at any time depends also on the prehistory and not only on the present input; such systems' input-output relations should not be confused with brain functions of individuals which are far more complex and subtle. Even though analogous oscillations are found in physics – and we refer frequently to them as analogies – it is definitely not meant that the actors behave mechanically.)

Hereafter a simple model for describing wave-like phenomena in sociology is proposed that can – beyond the application to simple master examples – also serve as a platform to describe more complex periodic processes. In view of the intrinsically very approximate character of the approach it is necessary to keep the level of mathematization as low as possible, and to emphasize functionalities rather than quantitative measures. It is of immediate relevance that the parameters can be sociologically interpreted and can serve in more complex cases as bricks of a semi-quantitative starting ground. The last part of the paper deals – notwithstanding the advanced mathematics in solving the relevant differential equations – in an even more qualitative sense, with initial phases of periodic processes which define the amplitudes of oscillations, as well as with the usefulness of equivalent circuit representations. Possibilities of extensions of the model are indicated.

2. Oscillations in Science, Nature and Society

Before we come to the discussion of relevant sociological issues, let us give a reviewing account of oscillatory processes on a more scientific level. In this section we will avoid a mathematical description but it is still necessary to make use of technical terms the readers might not be familiar with. (It may be noted that terms like cyclic behavior, wave-like behavior and oscillatory behavior are more or less used here synonymously as we deal primarily with harmonic periodic processes.) The necessary explanations and continuative literature can be found in the Supplement 1 and 2. The readers whose primary interests lie in the author's descriptive modelling set out in the main text may skip this section, at least in a first reading.

In spite of the time inexorably progressing, in other words in spite of the irreversible character of nature and culture, there are processes that are approximately cyclic. It is not so much surprising for particularly designed and particularly isolated, hence "artificial" laboratory processes, but it is so for natural processes. Cyclic processes are not only known in physics, chemistry, biology but also in economy or sociology. In technical terms, the configurational space spanned by the parameters (such as particle concentrations in chemistry) exhibits in such cases closed loops around a stationary point (see Supplementary 1) which may be the equilibrium state (zero entropy production) or a (dynamic) non-equilibrium steady state (non-zero entropy production) characterized by Prigogine's principles (Supplementary 2). In reality there is always an instability leading either to a decay towards equilibrium or to another non-equilibrium stationary state. It is worth mentioning that in real life "equilibrium states", states of zero entropy production typically involve frozen structure elements, they are then non-equilibrium states but not fully dynamic non-equilibrium states (see Suppl. 2).

A classic example of an oscillating system is provided by an electric oscillating circuit (ALEXANDER and SADIKU 2004) manifested by an LC resonator (coil with inductivity L and capacitor with capacity C connected in parallel) which shows harmonic oscillations on small excitations. Unlike for a resistor, in the case of an inductor the (negative) time derivative of the current (and not the current itself) is proportional to the voltage, while for a capacitor the time derivative of the voltage is proportional to the current. The electric balance leads then to harmonic oscillations of current and voltage around the (current-less) equilibrium state with a phase shift of 90° between each other. The amplitudes of the oscillations are determined by the initial excitation.

This holds for an ideal circuit without resistance (R); otherwise dissipation (friction) occurs, which leads to a continuous decay towards equilibrium. If oscillations occur, they are then damped with steadily decaying amplitudes. Also, large excitations lead to a different behavior, as here non-linear processes arise, formally characterized by voltage dependency of the parameters (L, C, R).

Mechanical analogues to the electric circuit are provided by a swinging pendulum or a vibrating spring (LANDAU and LIFSHITZ 1969). In the latter case the capacitance is replaced by the inverse spring constant and the inductance by the mass. Also here, harmonic motion requires absence of friction (mechanical resistance) and small excitations (proportionality between elongation and back-driving force). Frictions lead to damping and finally zero elongation; high excitations may lead to chaotic motion.

Note that the use of the term "equilibrium" for the stationary point is not exact in terms of thermodynamics, as from the standpoint of chemical thermodynamics the setup itself is a frozen construction. Chemical systems or more generally systems the kinetics of which are characterized by master equations implicitly include (besides capacitive elements) resistive elements (MAIER 2023, Suppl. 1). For small excitations we refer to the regime of linear irreversible thermodynamics. Then it can be shown that steady states can occur which are – unlike the equilibrium state – not characterized by a zero entropy production but by a minimal entropy production (Suppl. 2). Though out of equilibrium, such states show, like the equilibrium state, stability. Perturbations lead to a backdriving response, (in technical terms: entropy production is a Lyapunov function) and oscillations are impossible (Suppl. 2). They are however possible far away from equilibrium. Here only that part of the entropy production variation that is due to forces has to vanish for a steady state (provided certain boundary conditions as specified in Suppl. 2, are fulfilled) and the system can go through the same states multiple times. A necessary condition is that a kinetic potential does not exist, i.e. the variation of the force-related change of the entropy production is not integrable (more details are given in Suppl. 2).

If one excludes outer influences, the omnipresent dissipation has to be counteracted intrinsically in order to maintain or even generate the cyclic behavior. In electrical systems this may be caused by negative (differential) resistances, in chemical systems by autocatalytic processes (GLANSDORFF and PRIGOGINE 1971). Autocatalysis means that the reaction rate is enhanced by the reaction products. This corresponds to a positive feedback (close to equilibrium reaction products enhance the backward reaction rate and thus always provide a negative feedback) and hence to "negative differential reactivities" (MAIER 2019). The Zhabotinsky reaction is a famous example (PRIGOGINE 1980), another one the oscillating Hg-heart. Already in the latter example, transport processes are critically involved (MAIER 2023), their presence can lead to spatial oscillations and periodic patterns. The Liesegang phenomenon is another well-known example; here periodic spatial solid-state patterns are formed (WAGNER 1950). Ingredients such as positive feedbacks are omnipresent in living matter and particularly obvious in the fact of reproduction.

Biological processes such as the prey-predator interaction, where any large entity (the animal) is produced (reproduced) or annihilated (dying) *as a whole*, are also described by rate equations analogous to chemical reactions (FRANCOISE 2005, ISTAS 2000, KORPIMÄKI and KREBS 1996); then the same remarks as above – in particular to built-in resistive elements and their counteraction – hold. An appropriate example is the Lotka-Volterra system (KOTOMIN and KUZOVKOV 1996). TURING used coupled reaction-diffusion equations (TURING 1952) to explain pattern formation in biology. GIERER and MEINHARD interpreted periodic thorn formation (e.g. roses) or fur markings (e.g. zebras) by such mechanisms based on short range activation and long-range deactivation (MEINHARD 1974). HODGKIN and HUXLEY pioneered on a non-linear model of nerve excitation and propagation which needs to be mentioned in this context (HODGKIN and HUXLEY 1952). For more recent work the reader is referred to e.g. LECHLEITER et al. (1991), HONMA and HONMA (2003) and WILHELM (2009).

Prominent examples of oscillations in societal contexts are business cycles in economy characterized by sequences of phases of prosperity and recession (rising and falling market, ups and downs). An early explanation has been given by SCHUMPETER (SCHUMPETER 1961). A more sophisticated analysis based on the superposition of long- and short-term cycles plus fluctuations was called Schumpeter's clock (GOODWIN 1951) and analyzed in detail by MENSCH (MENSCH 1979), WEIDLICH and HAAG (WEIDLICH and HAAG 1983). Long-term cycles caused by innovations had been considered earlier by KONDRATJEW (KONDRATJEW 1926) and named after him. Cycles of supply and prices in livestock market (pork cycles, also named hog or cattle cycles) are typical short-term waves that find a simple explanation in the significance of a time-delay (see Suppl. 3) (ROSEN et al. 1994). The advantage of deterministic chaotic models over stochastic models in describing various business cycles has recently been discussed in ORLANDO et al. (2020).

The main text gives more examples of sociological processes of that type. MCCAFFREE describes oscillations of group size of animal or human entities (fission-fusion oscillation) that can drive sociological progress. Here the term oscillation includes periodic processes as a consequence of adaption to varying outer conditions and not processes as a consequence of inherent dynamics (MCCAFFREE 2022). In addition, abrupt transitional phenomena such as rebellion or chaotic behaviors in both economy and sociology show similarities to synergetic phenomena in natural science (HAKEN 1983). In this context the reader is also referred to more recent literature dealing with social phase transition (ZHANG et al. 2024) and the significance of multistability in biology (WILHELM 2009): To avoid the reproach to social physics of being too naïve: Processes that are dominated by personal decisions do certainly not generally follow simple rate equations or thermodynamic relations. But there are *collective* processes that show related characteristics and hence should be describable by *analogous differential equations*.

If the physico-chemical treatment is applicable, one can distinguish between two different ways to handle rate processes, one is to express the rates as driven by the deviations from the stationary states, the other to differentiate between forward and backward reactions and to treat each as functions of the number densities (concentrations) of the reaction partners. The justification of using such rate equations in our context is based on the fact that they describe encounter probabilities.

Chemical processes are generally rather accurately described by rate equations, as here particles are generated or annihilated (collision models). The application to biological processes is naturally less precise as here complex beings are involved, but they may still do a good job if the *total entity* under consideration is – as a consequence of the encounter processes – generated or annihilated as a whole and hence behaving as a "superparticle". In the context of societal processes, the application of such equations has to be taken with an even much greater grain of salt as here intellectual feedback loops play a prominent role. Modelling by using analogues from natural science can only be of use for crowd processes and then if applicable only on a very approximative level.

In all cases, it is pertinent to distinguish between forced oscillations (outer periodic "forces", e.g. adaption to supply cycles imposed by fashion industry) and free oscillations. The first drive a system that is able to vibrate into stationary oscillations whose frequency coincides with that of the force. Contrarily, in the case of free oscillations (on which we will concentrate in the following), such persistent outer force is absent but the oscillations may be *triggered* from the outside (forcing initial elongation of the pendulum, applying a voltage pulse, or triggering sociological cycles by the prehistory, see main text); they can also be self-excited (e.g. by virtue of a negative resistance element (SCHÖLL 1987), by autocatalysis or self-amplified spontaneous action). The latter is an important element of structure formation in all branches of science. Trigger processes set the initial conditions and are crucial for the amplitudes of the caused vibrations.

As the degree of quantifiability becomes increasingly lower when we proceed from hard to soft sciences or better when we increase complexity, over-mathematization and over-parametrization must be avoided. For this very reason we will use here a most simple, heuristic level of description as used e.g. in electrical circuits or mechanical springs referred to in the beginning. It turns out that this leads to a meaningful, yet rather general platform with a minimum number of interpretable parameters useful for the description of a great number of oscillatory processes in sociology. Such platform can be broadened, deepened or detailed on demand.

3. Significance of Memory

What makes sociological problems very different from simple atomistic ones is the fact that memory effects play an important role. Here it is not so that encounter processes or momentary assessments lead ad-hoc to defined results, rather decisions are based on the judgment of an actual situation on the basis of experience (prehistory).

Note again that memory-effects in the overall behavior of the system (crowd) under concern have to be understood in the sense of system theory (the overall output depends not only on the input at present time but also on prehistory) and should not be confused with the brain actions of individual persons (which are characterized by a great complexity and subtleness) even though it depends on them. The fact that – in some examples of interest – we can deal with collective properties rather than with simple statistics of complex isolated personal decisions lies in the nature of social interactions. The scope of this paper is not to describe these interactions, rather to take the crowd behavior as an empirical fact and describe it by "macroscopic" differential equations and parameters. In technical terms, we are concerned with collective properties treated in a "mean-field" sense.

It is noteworthy that it is – in the context of our model – memory and not complexity of the mechanism which eventually leads to oscillations under otherwise weak assumptions. In order to exemplify the effect of memory, we consider, as a preparation, a simple collision model that we later even directly apply to describe fashion cycles (see also Supplementary information 4). It serves as a first step of our consideration but is already very revealing in this context. The results are shown by Figure 1.

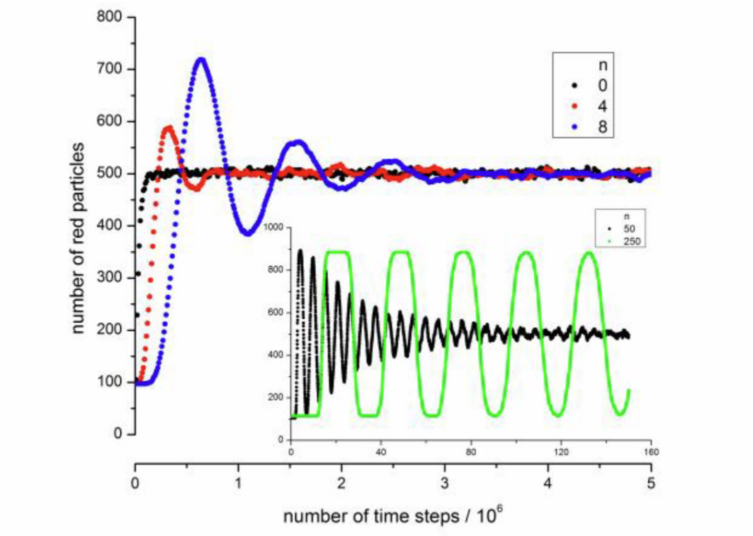

Fig. 1 Monte-Carlo collision experiments with memory (n). Unlike the memory-free case (n = 0) for which one observes an exponential relaxation, the memory effect leads to a damped oscillatory behavior. The model assumes that a blue particle that has experienced a converter contact for the n-th time changes into a red one vice versa. (Initially: 900 blue particles, 100 red particles. The number of catalysts is 9000.) Cf. also Supplementary 4.

In this collision model a blue particle whenever meeting a "converter" transforms, in the memory-less case, into a red one and vice versa. Besides color, the particles are a priori identical. Starting with a non-equilibrium distribution, in the memory-less case the expected exponential relaxation towards the situation of equipartition ("equilibrium") takes place. Now, a memory effect can be included by the rule that a blue (red) particle has to meet the converter not only once, but for n-times before it changes colour. As obvious from the figure such introduction of memory leads to a damped-oscillatory behavior which is the less damped the greater *n*. (Note that the evaluation of the so-called logistic curve leads to similar responses if time-delay is introduced (May 1973, Forrester 1968, Berg and Kuhlmann 1993)).

As long as the concentration of the converter is large and thus approximately constant, the described memory-less reaction is – from the standpoint of collision kinetics – a pseudo-first order reaction showing an exponential time-behavior. Such a behavior can be recast in terms of a proportionality between reaction rate and deviation of actual value from equilibrium value as driving force (see Suppl. 4). In the case with memory, we refer to a sequence of pseudo-first order reactions as far as the individual reaction partner is concerned.

In order now to describe processes that are more general than encounter processes but exhibit memory effects, we will make the *thermodynamic ansatz* that the reaction rate is proportional to the time integral of the deviation of the actual value from the realistically expected value, i.e. to the time integral of the driving function. (For the conceptual connection with the collision model see Supplementary Information 4). If only the momentary estimates were decisive, the rate would be given by the actual deviation only. Whenever equivalent collision models are not simply of first order, this *ansatz* appears most reasonable for small deviations, while for large deviations the parameters hardly stay constant.

Such procedure could be directly applied to fashion cycles treated in Suppl. 4: If we restrict ourselves to the mechanism of assimilation and consider the distribution of two colors, we can assume as major driving force the deviation from equipartition of the two colors, an established fashion, once having become popular, shows the tendency to be getting outdated until the memory of its high-time has faded away. (Note that we are not considering adaption to outer "forces", e.g. forced oscillations imposed by fashion industry.)

The driving function can be generalized, and thus also be applied to the difference of actual, expected value and true value of a property. This will have impact on a sociologic activity that tends to diminish this difference. If we finally allow for taking account of the fact that this activity can also change the true value, we arrive at our general model described now and detailed in the next sections.

A relevant master example (later we will address further examples) may be the introduction of an innovation that had initially been "oversold". After having realized that the expectation does not match reality, the interest decreases. In the memory-less case this decrease will be monotonic; but in the case of memory this decrease will – owing to memorizing the better past – occur in a decelerated fashion but in turn then for the very same reason be eventually stronger than the innovation deserves. This frustration phase in which the innovation is undervalued may then trigger the next overestimation phase.

This pattern is met in various fields: in science (consider the initial flood of publications in temporarily hot topics the interest in which (and so the number of publications) will – after a while – inadequately shrink), in technology (initial down-playing of the importance of a promising technological innovation now may lead to a hype) but also in politics (the sudden breakdown of a political systems may cause overestimation of advantages of the competing system, or to give another example, an overrated newly elected political leader may soon perceive frustration effects in the next elections). The processes to be described here are such that the estimated value of a given property (i.e. the prognosticated or expected value, E) deviates strongly from its real, true value (e). After the constraints responsible for the initial valuation are removed, the system is prone to oscillate. In realistic situations the oscillations may become highly damped or may already after one cycle reach a plateau (cf. Gartner hype cycle (FENN and RASKINO 2008)), what can be ascribed to the impact of additional irreversible processes, e.g. enabled by learning effects (PEARTON 2007).

In this contribution we are primarily interested in the underlying aptitude of a sociological system to oscillate and emphasize that this is a consequence of typical human behavior (connected with memory, experience, and inclination of being persistent) that endows the system with the necessary inertia, without which a spontaneous response would occur. In the beginning, thus, we will abstract from the initial phase as well as from effects that lead to damping or to other realistic modifications. The model to be outlined – in view of the very approximate character – will be kept as simple as possible and hence being characterized by a minimum of sociologically important parameters. Even though our main interest is devoted to the oscillation phase, we will later shortly deal with the initial phase setting the initial condition, not only as this prehistory determines the amplitudes of the oscillations but also since important sociological scenarios such as dictatorship in terms of exclusivity of power or of information as well as tabooization (caused by external or internal mechanisms) can be addressed in this way. Within our toy-model such a pre-phase is formally most simply modeled by considering the evolution of the sociological parameters into stationary values.

While such parameter variations could also account for learning effects and hence for realistic cycles, the obvious isomorphicity of the sociological model to be described with oscillations in physical processes (free oscillations of a spring, or an electrical circuit consisting of capacity and inductivity, after having removed the bias) suggests damping to be most easily modeled by introducing a direct response channel into a *sociological equivalent circuit*.

Let us characterize our model more precisely: (i) It considers as "driving function" (Δ) for the changes in sociological activity (a) the difference between *promised value* (E) and *real value* (e), or in general but less precise terms the discrepancy between *seeming* and *being*. (ii) It considers the *time integral* over this "driving function" (and not Δ directly) as proper driving force being proportional to the time change of the activity representing the reaction rates. (Note that a may e.g. represent the number of scientists or the number of publications dealing with a novel scientific finding, or money that is invested in the context of a novel technology).

4. The "hype-problem" and the Role of Accumulated Discrepancy between Expectation and Reality

For clarity's sake, but without significant loss of generality, we consider the following master example: A new technology is appraised and a certain, initially oversold success proclaimed (E) exceeds the real (or true) success value (e). The difference $\Delta \equiv E - e$ may be called here the exaggeration gap. The relation between E and e is characterized by the parameter $\varepsilon = \partial E / \partial e$. (If the influence of other variables such as a is implicit through e, then E is a unique function of e and $\varepsilon = dE / de$.) The promised success will prompt activities contributing to the success of the technology. The fact that it is $\int (E - e)dt$, i.e. the accumulated discrepancy between expectation (or more generally the apparent value) and reality (real value) that drives, or more precisely will diminish, the reaction rate ($\dot{a}$) in a multitude of problems, agrees with daily life experience. It is not the immediate deviation of E from e that will trigger changes in the activity. But if this discrepancy is perceived over a certain time interval, this will happen.

The second necessary parameter links E and a through $\eta = \partial a / \partial E$. (If a is a unique function of E we can replace the activity ($\dot{a}$) change by $\eta \; d/dt \; E$). Frequently η can be considered approximately constant; then the activity will linearly increase with the expected success. (In fact, in many realistic examples (see below) E will just be measured by a directly so that η might simply be set to unity.) Examples for a may be the number of enterprises, of employers, of publications dealing with a topic of promised success E.

These assumptions naturally lead to (t: time, dot: time derivative):

$$\dot{a} = -\alpha \int_0^t (E - e)dt + \dot{a}(t = 0) \tag{1a}$$

or for time-invariant α, η

$$\ddot{a} = -\alpha(E - e) = \eta \ddot{E} \tag{1b}$$

The simplest problem is characterized by a constant e-value ($\hat{e}$) (i.e. ε being infinitely high). Then the second derivative of E in Eq.1 can be replaced by the second derivative of $(E - e)$ leading to a harmonic oscillation problem with oscillations of E around $\hat{e}$ as shown in Fig. 2 (ω: angular frequency). Note that then a is in phase with E, but the rate $\dot{a}$ is out of phase with the driving function $E - e$ (cf. Suppl. 5 for more details). Examples where the assumption e = const is fulfilled will be tackled later.

More generally however, the realistic success e will be a function of the activity. (In terms of the example of a hyped new scientific finding, the number of scientists turning to the new field (a) will not only affect the driving function, but also directly influence the real success). The reference value $\hat{e}$ will then only be met for the corresponding activity $\hat{a}$. If we choose $\hat{a}$ such that $E(\hat{a})=e(\hat{a})$ ($\equiv\hat{E}=\hat{e}$) then we refer to the expectation "equilibrium" value. We will adopt this choice for the following. Here oscillations can occur if the deviations of e and E from $\hat{e}$, $\hat{E}$ (denoted by δe and δE) are proportional to each other what corresponds to a constant ε – parameter. If $\hat{e} = \hat{E}$ and thus $E - e = \delta E - \delta e \propto \delta E$, the differential equation simplifies to $-\ddot{E} = -(\delta E)^{\cdot\cdot} \propto \delta E$, with sinusoidal solutions for e, E oscillating around $\hat{e}$ (= $\hat{E}$) as in the previous example (cf. Suppl. 5). The activity a will oscillate around $\hat{a}$.
If $\hat{\Delta} \equiv \hat{E} - \hat{e}$ is non-zero, it is straightforward to show (s. Suppl. 5) that then δE oscillates around $-\frac{\varepsilon}{\varepsilon - 1}\hat{\Delta}$ and hence E – but also e – will oscillate around $\frac{1}{\varepsilon - 1}(\varepsilon\hat{e} - \hat{E})$. The parameter a will oscillate around $-\frac{\eta\varepsilon}{\varepsilon - 1}\hat{\Delta} + \hat{a}$. (The previously considered case of constant e is reproduced for $\varepsilon \to \infty$ where then E oscillates around $\hat{e}$ (=e) .)

The simplest case is met for $\hat{E} = \hat{e} = 0$. Then E and e oscillate around zero. The phases where both are negative can be interpreted as phases of failure. Whenever $e = 0$ then it is intelligible that also $E = 0$; as far as the activity is concerned, the curvature will be zero, if not the absolute value itself. See Fig. 2 for details.

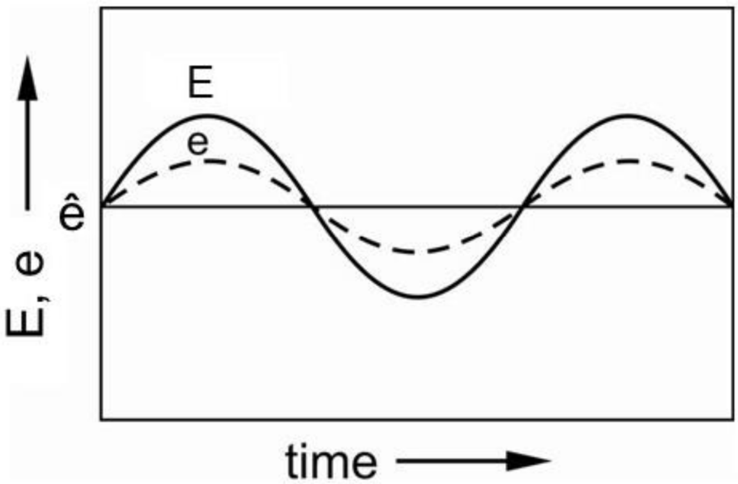

Fig. 2a Oscillations of expected and real success ("seeming and being") around the "equilibrium" value ($\hat{e}$). In the simplest case $\hat{e} = 0$ and the phase of negative values is to be interpreted as phase of failure. In the most general case where $\hat{e} \neq \hat{E}$ the central value $\hat{e}$ has to be replaced by $(\varepsilon\hat{e} - \hat{E}) / (\varepsilon - 1)$. If e is kept constant (i.e. $\varepsilon \to \infty$), only E (and $\delta a/\eta$) oscillates around $\hat{e}$. The "equilibrium" state ($\hat{e}$) is characterized by identity of expected and real success ("seeming equals being").

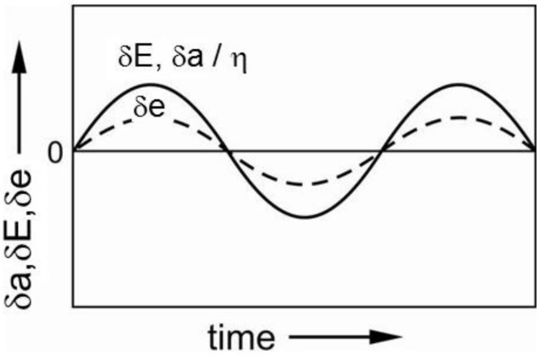

Fig. 2b Oscillations of the deviations δE, δe, δa from the stationary value which is zero for $\hat{E} = \hat{e}$ as shown and $-\hat{\Delta}\dfrac{\varepsilon}{1-\varepsilon}$ otherwise (not shown).

The interpretation of the harmonic behavior is as follows: A gap between expected success and real success and consequently an over-activity characterizes the over-motivation phase. This gap does not directly affect the rate $\dot{a}$ but rather the acceleration $\ddot{a}$. Only after this gap has continually increased for a while, the activity will decrease. In other words, there is a certain credit that needs to be exhausted. This then leads to the pendulum swinging into the other direction. The over-motivation is replaced by frustration and now the success is judged too negatively. The real success is comparatively larger, but it takes again some time until this is counter-acted. It is worth reiterating that such a simple situation would not lead to oscillations if $\dot{a}$ were proportional to $(E - e)$ directly, rather a would – depending on the sign of $(E - e)$ – exponentially decay or continuously increase. It is obviously the described inertia that is necessary for the addressed oscillations.

There is however a class of phenomena that although based on $\dot{a}$ being proportional to Δ, shows oscillations, namely if a time-delay τ is decisive leading to $a(t)$ being proportional to $\Delta\ (t \pm\tau)$. Examples are the pork cycles in economy where the supply is based on the assessment of the demand at different time. A sociological example are students deciding for a certain subject on the present need and do not realize that it is the need after the graduation which is the relevant measure (see Suppl. 3). We do not continue to consider such quite obvious cases, but rather come back to our toy-model. All the parameters (they are listed in table 1 and interpreted in the next section) are taken as time invariant (but see below) corresponding to delay-less relations between a, e and E. The relation between a and E had been characterized by the parameter η which when constant is given by $\delta a / \delta E \equiv (a - \hat{a}) / (E - \hat{E})$, while the relation between E and e had been characterized by $\varepsilon = \delta E / \delta e \equiv (E - \hat{E}) / (e - \hat{e})$ if ε assumed to be constant. (To reiterate: in this definition ε becomes infinitely large for our first case where e stayed constant). The third parameter α is defined by Eq.1 and refers to the negative ratio of rate change and driving function ($= -\ddot{a}/\Delta$). The combination $\alpha\ (1{-}1/\varepsilon)/\eta$ is identical to the square of the angular frequency (ω) of the oscillations. Note that for the oscillations to occur it was not necessary to assume $\hat{E} = \hat{e}$, but there are cases where this simplifying assumption is reasonable ("expectation equilibrium"). The most simple case is characterized by $\hat{e} = \hat{E} = \hat{a} = 0$, the parameters then refer to the ratio of the absolute values of E, e, a which are hence proportional to each other. In these cases, $\alpha\ (1{-}1/\varepsilon)/\eta$ and thus ω^2 gives the ratio $-\ddot{a}/a$.

But back to the general case: If we draw the formal analogue to the electrical situation, $-d\ddot{a} / d\Delta = \alpha$ corresponds to an inverse social inductance (L) whereas $da / d\Delta = \eta\varepsilon / (\varepsilon{-}1)$ corresponds to a social capacitance (C). Both elements are connected in parallel as they refer to the same driving function Δ. If the model parameters are constant, as assumed in this contribution ($C = \eta\dfrac{\varepsilon}{\varepsilon-1}$ and $L \equiv 1/\alpha$), the circuit parameters C and L are constant too and oscillations are observed with the angular frequency $\omega = 1/LC$ (Suppl. Fig S10).

If we refer to the mechanical analogue of a vibrating spring (mass m connected by a spring of spring constant D with a rigid wall), L would have to be replaced by m (inertia), and C by $1/D$ (softness). This comparison may even be more meaningful as inertia is a more straightforward analogue than magnetic induction. More complex mechanisms (e.g. of Lotka-Volterra type) can easily introduce phase shifts between a and Δ, or weighting functions may be introduced by replacing the integration by a convolution (GOMATAM and MACDONALD 1975, ARDITI et al. 1977). We will return to the latter point at the end. At the moment, however, we want to keep the description as simple as possible. As with any model, the information with respect to specific situations is hidden in the parameters, here ε, η and α. Before we inspect them, let us (i) consider examples for the most simple case characterized by e = const, but also (ii) investigate as to what conclusions other, more complicated non-linear $e(a)$ functionalities would lead.

The simplest case of a constant success value has already been addressed. (As mentioned there, this special case follows for $\varepsilon \rightarrow \infty$ which simplifies the social capacitance to $C = \eta$.) A first example where this may be approximately fulfilled is a rather saturated technology in the case of which the real success (in contrast to E) is not sensitively altered by varying activities. In this case we get oscillations of E and a but not for e which is always given by an "equilibrium value".

A second example refers to a related class of problems viz. the fashion problems (cf. Suppl. 4). Fashion does not only concern clothing, naming is another example besides many others. The popularity of given names is one of the few cases of interest where reliable statistics are available. The ranking of given names can be followed for Germany for the entire last century. Figure 3 shows two examples, referring to a male and a female name the popularity of which initially decreased presumably owing to excessive use, and then experienced a revival. The mapping of this problem on our master example is evident if we identify E (or a which we do not need to distinguish here) with the popularity of the name measured in terms of its actual frequency of occurrence, i.e. taking account of the sociological context ("apparent" or "perceived value"), while $e = \hat{e}$ is a measure of how appropriate the name is, would one disregard the incidence ("true value": What would be the name's preference if one did abstract from its current popularity?). For simplicity we concentrate on the exclusivity effect as the major driver: If the name has become exceedingly popular there is a tendency not to use the name. Owing to the memory this continues to be the case initially even for E being smaller than e, until the accumulated rareness of the name increases its use again. The half-period of about 50 years suggested by the figure is a typical period within which an outdated given name got sufficiently out of "sight". (As outlined in the Supplementary S4 and referred to at the end, such problems can also be adequately treated by collision kinetics with memory.)

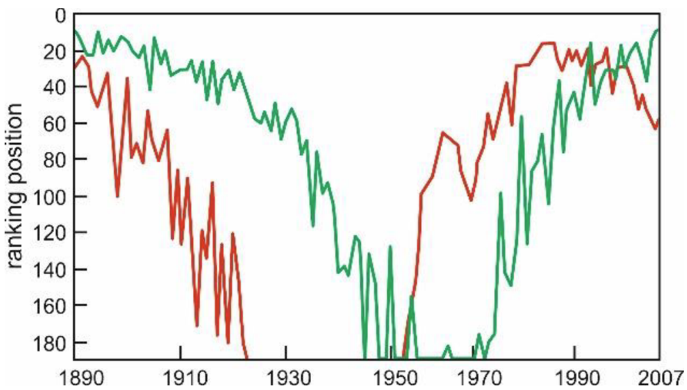

Fig. 3 Frequency of the given names "Max" (green) and "Carolin(e)" (or "Karoline") (red) in Germany for the last 120 years. Decline and revival of the names can be directly explained by the exclusivity effect. (See also Suppl. 4.) *Reproduced with permission of knud@beliebte-vornamen.de.*

Now let us inspect some non-linear $e(a)$ functionalities. For simplicity's sake, we set the proportionality factors in the differential equations to unity and additive constants to zero. We concentrate on powers in $\Delta \propto a^n - a$ other than $n = 0$ or $n = 1$. The mathematical solutions soon become extremely involved. Here it may suffice to make only a few remarks. If $0 < n < 1$, a^n exceeds a for $a < 1$; then a increases in an accelerated way allowing for one-sided oscillations (see Supplementary 6), i.e. the inflection point around which the function may oscillate is generated on the positive side. For $a > 1$, a^n is smaller than a, and is increasingly so for greater t, leading to deceleration and non-monotonicity. For $n > 1$ the situation is inverse, first characterized by a deceleration and then – if a exceeds unity – by ever increasing acceleration. The latter then leads to a catastrophic behavior.

Typical solutions for $n = 2{,}3$ are based on tangent or cotangent functions; they show a periodical but not harmonically oscillating behavior (see Supplementary 6). The diverging behavior contained in these solutions makes the full-time evolution unrealistic but highlights the explosivity of such a situation. (The term "unrealistic" implies that in such situations the parameters will certainly not remain invariant). At any rate, the behavior in the first period is intriguing. The steep increase of a, E, e near the asymptotes means that the impact of this technology steeply grows as may be the case for a very successful young technology, which has the ability to easily outcompete others.

An interesting question that may arise here is how sociological systems (or technologies) compete with each other. In the literature, there is an adequate kinetic model of how molecular species compete about common limited resources (EIGEN 1971), which led to an epitomization of DARWIN'S theory. Setting up a similar sociological theory by implementing memory effects as done above would be a very intriguing challenge but is beyond the scope of this paper (see also Supplementary information 7).

5. Model Parameters

Let us return to the initial functionalities and discuss the meaning of our model parameters (see table 1). In the above defined terms α is a measure of how strongly the activity is accelerated by a certain integral discrepancy between promise and reality ($1/\alpha$: "degree of inertia"), it is a measure of reaction readiness, short-windedness of the system. Most generally formulated, α plays the role of a reciprocal *resilience factor* and is proportional to the square of the frequency (ω) of the oscillations $\cong \sqrt{\alpha\left(1-\frac{1}{\varepsilon}\right)\frac{1}{\eta}}$. Low frequencies indicate a sluggish reaction, the actors are rather inert. Very high frequencies indicate a hysterical behavior. Examples can range from quick responses by hectic persons to collective cultural movements that involve generation changes.

Also the other model parameters η and ε are contained in ω. As in many cases E is simply measured in terms of a, the meaning of η is then trivial. Generally speaking, it represents a conversion or *permittance factor*. In our master example the η-factor characterizes how strongly the promised success translates into activity and may also termed motavitability.

Finally, ε, i.e. the ratio $\delta E/\delta e$ characterizes the hype most directly and may be termed *imbalance factor.* If this factor is a constant close to unity and $\hat{E} = \hat{e}$, one deals with extremely careful and conservative estimates of the situation $(E \simeq e)$; $\varepsilon = 1$ characterizes a completely realistic judgement and conservation of a stable expectation "equilibrium" ($\omega = 0$). As the above considerations have shown, ε need not to be constant to enable oscillations but constancy is, as far as its role in Eq. (1b) is concerned, sufficient. Constancy of ε is best fulfilled for small excitations, i.e. for not too large a discrepancy concerning expectation and reality. In cases where the real success value is more or less fixed, distinct changes of E demand very large ε-values, as already discussed for saturated technologies; otherwise δE is also very small. It is interesting to note that the product of η and ε characterizes the sensitivity of e with respect to $a,$ and vice versa.

As we take all these parameters as time-invariant, in our simple model an important qualitative characterization is the following "aphorism": *Whatever is responsible for the pendulum swinging too much to the right, is also responsible for its extreme swinging to the other direction.* In the same sense then: One does not do any good on a long run for propagating an idea or a concept if one is overstressing its advantage; the almost unavoidable backslash will punish the exaggeration. Except the fact that probably a certain Δ is needed to overcome initial "nucleation barriers" (e.g. also information barriers), one is well advised to stay as close as possible to the "expectation equilibrium".

How great the amplitudes of the oscillations are, is determined by the prehistory (cf. bias of a mechanical or electrical oscillatory system). A complete theory that includes prehistory would explain the evolution of the harmonic oscillations by the inner dynamics of the overall system. As this has to involve specific information on the specific case, such an attempt would be beyond the scope of this paper. A few remarks may suffice.

In concrete or abstract terms, the initial situation is dictated by the initiating forces (analogous to the force that extends a spring). A relevant pre-phase may consist in the introduction of an innovation. In such a pre-phase Δ may be dictated by the exclusivity of information that e.g. a company possesses when announcing an innovation before the true value can be tested. Here let us take a purely heuristic route and let us come back to the question of initial conditions by considering the physical master example of a vibrating spring as analogue (without suggesting that the sociological ensemble has direct similarity to a mechanical system) which is characterized by the isomorphic differential equation. The frequency is fixed by the force constant and mass of the spring, but the amplitude determined by the initial condition ($\sqrt{a^2(t{=}0) + \dot{a}^2(t{=}0) / \omega^2}$). Such an initial condition can be characterized by a force compensating the spring force that is suddenly released but also by a sudden variation in the spring constant (or the mass) in the pre-phase (LANDAU and LIFSHITZ 1986).

A simple case of a pre-phase setting the initial conditions for a is a phase in which the activity is suppressed or kept at a low level for political reasons (dictatorship) or for reasons of "political correctness" (taboo-phase) (WEIDLICH and HAAG 1983). After overcoming this pre-phase the system undergoes free oscillations with an amplitude set by this pre-phase. Another example may be the deliberate downplaying of the success expectation.

Unlike materials objects met in oscillating circuits such as springs, coils, capacitors etc., in sociological systems the parameters (α, η, ε) are sociologically determined. In the framework of our simple model, we thus characterize the pre-phases by artificial settings of these parameters followed by a relaxation towards stationary values (now termed α_∞, η_∞, ε_∞ etc.). Let us briefly investigate two simple cases, with opposite effects on the development of frequency and amplitude.

One example may be that α is artificially set to zero (or the "degree of inertia" $1/\alpha$, i.e. the social resilience factor to infinity) and hence, even if a gap between E and e is realized, a counteraction (as in the case of an infinite mass) is impeded. This could be directly related with political issues, e.g. that in a totalitarian system the discrepancy between propaganda and real value may be perceived over a while but no collective reaction is allowed to occur. After suspension of the dictatorship-phase α may develop into the above stationary value, now called α_∞ – in the simplest case – with a time constant τ_α, according to $\alpha(t) = \alpha_\infty\left(1-e^{-t/\tau_\alpha}\right)$. (Another example is that a product is announced and oversold, yet counter-acting measures cannot be undertaken before the product is on the free market).

The second example is that in spite of a finite Δ, the activity is remaining on a low level owing to an artificially small value of $C \equiv \eta \frac{\varepsilon}{\varepsilon - 1}$. The sociological reason may be that in view of the sociological context people do initially not see the perspective to change the situation. In the last case we may – on the level of our toy-model – similarly formulate $C = C_\infty\left(1 - e^{-t/\tau_C}\right)$.

If we accept such most simplified time evolutions, we obtain the differential equation $\ddot{a} = -\frac{\alpha_\infty\left(1 - e^{-t/\tau_\alpha}\right)}{C_\infty\left(1 - e^{-t/\tau_C}\right)} a$. In many cases the two-time constants may be sufficiently different, so that the treatment can be decoupled. Assuming arbitrarily $\tau_C \ll \tau_\alpha$ the long-time solution based on Bessel functions and sketched in Figure 4 is met.

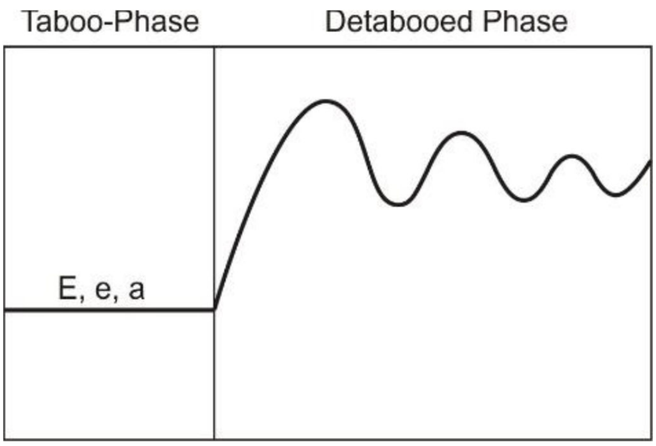

Fig. 4 Technology is politically incorrect in the taboo phase. Even though the expectation value of success may be high, there is no reaction in terms of a. In the de-tabooed phase, α increases from its (near) zero value to its equilibrium value α_∞. The oscillatory solutions are damped as derived in Supplementary 8.

The solution based on Bessel functions yield more and more weakly damped harmonic oscillations as time proceeds (with a damping factor $\propto t^{-1/2}$). The amplitude then converges to the value expected for α_∞, C_∞. More details are given in Supplementary 8a. If $\tau_C \gg \tau_\alpha$ the solutions are given by hypergeometric series (see Supplementary 8b) having related long-time properties. (These involved solutions have been shifted to the Supplementary on purpose in order not to suggest too great a degree of quantifiability. Only the functional form of the solutions there are of significance.)

An explicit sociological model would allow for a more accurate description of the above mentioned phase transitions and a more exact time evolution of the parameters. However, in view of the complexity of sociological reality, it is advisable to keep the mapping as robust as possible. Breakdown of what is termed taboo-phase in Fig. 3 can happen extrinsically (e.g. by external force) but also intrinsically. Typical kinetic phase transition models (WEIDLICH and HAAG 1983, HAKEN 2004) may involve fluctuations (around the dictated values) leading to instabilities by self-amplifications and finally to the collapse of the initial situation (cf. also Supplementary 7). Such specific considerations are beyond the scope of the present contribution.

Let us now discuss an example in greater detail for which empirical data are available. It refers to the popularity of painters in the last century as measured by "google-culturomics". MICHEL et al. (2011) compare the citations of Marc CHAGALL in the UK and in Germany in the 20th century. The data are given in Fig. 5. They attributed the difference to the suppression of this artist's appreciation by the dictatorship during World War II in Germany. Apart from a slight overall increase both curves show oscillations with a period slightly less than 10 years. (This may be understood in terms of typical fashion-like cycles (see Suppl. 4)). While the normalized behavior for the UK is rather well-behaved, in Germany (where the work of CHAGALL was suppressed in the 1930th and the war times of the 1940th), after World War II heavily damped oscillations are observed that develop towards the behavior in the UK. For a rough description we can consider e (realistic value) as being given by the normalized mean value. Then, a description according to the behavior just discussed is possible (cf. Fig. 3), where it is the overcoming of the depression phase that leads to oscillatory transients in the Germany example which are close indeed to the behavior discussed in Supplementary 8a $\left(\alpha = \alpha_\infty\left(1 - e^{-t/\tau_\alpha}\right)\right)$.

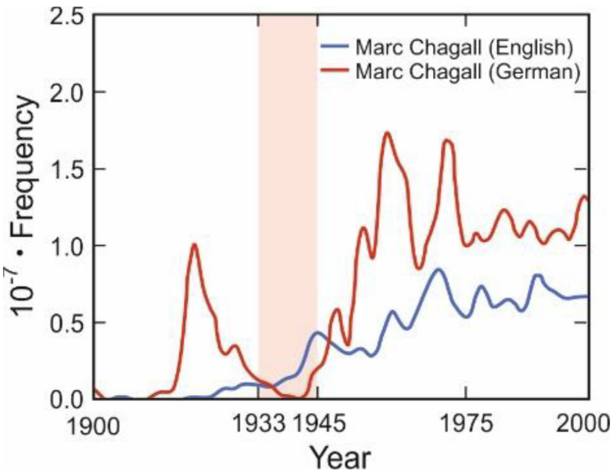

Fig. 5 Citations of Marc CHAGALL in UK and Germany in the last century as a function of time measured by Google-culturomics. *Reproduced with permission of The American Association for the Advancement of Science* (cf. also ref. 23).

Tab. 1 System parameters of the Model

Term	Definition	Meaning
$1/\alpha$	$-\int (E-e)/\dot{a}$	resilience factor
ε	$\partial E/\partial e$	imbalance factor
η	$\partial a/\partial E$	permittance factor

The table refers to the primary parameters of our toy-model. Derived parameters are $C=\eta\ \varepsilon\ /\ (\varepsilon-1)$, $L = 1/\alpha$ and the eigenfrequency $\omega = \sqrt{\frac{\alpha}{\eta}\frac{\varepsilon-1}{\varepsilon}}$. E refers to the expectation value and e to the real value of the specific problem.

6. Damped Curves, Realistic Cycles and Equivalent Circuits

In realistic sociological systems undamped oscillations are not met at least not for many cycles. We mentioned initially Gartner's hype cycle (PEARTON 2007) which shows that usually after a phase of enthusiasm, a frustration phase occurs which soon can lead to a plateau, named *expectation equilibrium* ($E = e$) in our model. Such a sudden equilibration can be reached by learning effects that have not been considered in our initial model.

This again can, however, be heuristically implemented by time dependent ε, η and α parameters what would then change amplitude (damping) and frequency. As an example, Supplementary 9a shows the effect of an exponentially decaying (motivability or more generally) permittance factor η leading to $\ddot{a} = -const\ e^{+t/\tau'} a$. Similar results are obtained by testing a hyperbolic decay in our toy-model (Supplementary 9b). The results yield already realistic shapes. Learning effects can also be taken account of by letting ε be a function of time. A quick learning could e.g. lead to a rapid decrease towards 1.

Let us inspect another simple approach, guided by the analogy with the oscillating circuits in physics, wherein we can mimic damping by admixing a direct response term ($\dot{a} \propto E-e$) complementing the *sociological equivalent circuit* to form a parallel L-C-R circuit (see Suppl. 10). The magnitude of the pre-factor would in electrical language be proportional to a conductance (i.e. inverse resistance $\equiv R^{-1}$). This value or better the ratio between this parameter and α decides upon the quickness of the response. This "direct", spontaneous social mechanism acts as a valve or leakage channel mitigating the initial discrepancy between expectation and reality (or most generally between seeming and being). If the latter is absent there is no damping; if it completely dominates, the situation will relax towards the expectation equilibrium without oscillation. This leakage channel need not correspond to a second mechanism, it can also be considered as correction to the assumption that it is the acceleration $\ddot{a}$ only that is proportional to Δ. As the accumulative response channel can be characterized by a convolution

$$\dot{a}(t) - \dot{a}(0) \propto \int_0^t f(\tau - t)\Delta(\tau)d\tau \qquad (2)$$

with $f \equiv 1$ (I-element in the language of system-theory), and the direct channel by the convolution with $f = \delta$ - function (P-element), one could formulate the general situation by a convolution $f * \Delta$ with f revealing P-I characteristics. In the language of system theory, we refer to admittance (impedance) that is characterized by resistive and inductive influences. (The capacitive relation $a \propto \Delta$ corresponds to a D-characteristic if $\dot{a}$ is taken as out-put signal.)

Note that the application of the convolution integral assumes linear and time invariant systems. For more general cases see e.g. ROEHLER (1973), MISHKIN and BRAUN (1961). Even though still heuristic, such approaches seem straightforward for explaining a highly damped curve such as the Gartner curve, and appear more natural than conceiving it as an artificial composite of two mechanisms with different time constants, viz. of a bell-shaped enthusiasm/disappointment response (i.e. as a curve that is ascribed to a quick emotional response) and a more sustainable logical s-shaped response as done in literature (FENN and RASKINO 2008). Here we would generally describe it as a mixture of direct and accumulative reactions, e.g. reflecting different mechanisms or differently acting social sub-groups (see Fig. 6), or alternatively – and this may be understood as a correction to assuming $\Delta \propto \ddot{a}$ – by an implementation of learning effects (learning from the previous phase) into an otherwise oscillating rigid system. It is worth emphasizing that learning effects (in contrast to the simple damped oscillation) can also take account of a time dependence of the frequency (Supplementary 9).

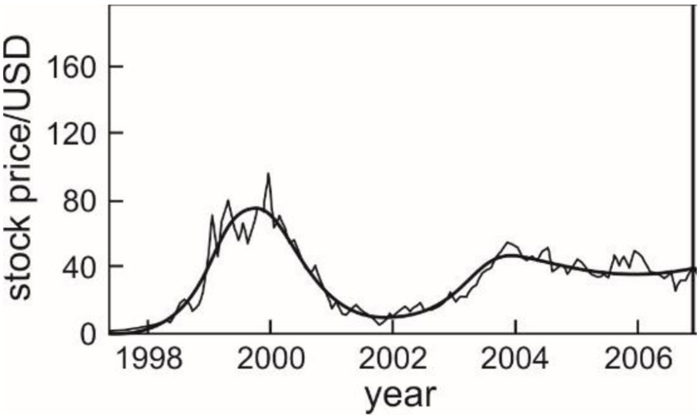

Fig. 6 The figure shows the chart plot of Amazon stock price from 1998 to 2007. (In 2007 another upward trend sets in). Realistic activity curves are rather characterized by damped cycles which can be described by superposition of indirect and direct responses or by learning effects. *Reproduced with permission of Yahoo! Inc. ©2011 Yahoo! Inc. YAHOO! and the YAHOO! logo are registered trademarks of Yahoo! Inc.*

Let us state again: The simple L-C model that is in the foreground of this contribution does not include non-reversibility. In the framework of this model, non-reversibility can be introduced by making the model parameter time dependent. Alternatively, one can introduce non-reversibility by admixing resistive (i.e. dissipative) effects (L-C-R model) and/or additional non-linear circuit elements. Just the opposite path is taken by the third approach that was already touched upon in the very beginning, where we used collision kinetics. This collision kinetics is per se far from a reversible behavior and oscillations are damped (cf. Suppl. 3). An oscillatory behavior was arrived at by introducing a memory through the trick that collisions have to take place several times in order to be efficient. Let us refer again to fashion cycles, but now treat it in terms of kinetics. (This tool was already mentioned in the very beginning of Section 5). It may be worthwhile to repeat that we do not intend to describe the adaption to forced outer effects (forced oscillations by periodicity imposed by fashion industry.) We consider a carrier of blue cloth who on meeting another carrier of blue cloth is stimulated to change to the alternative red color ("exclusivity effect"). Neglecting other "reactions" (such as blue (B) converting to red (R) when meeting red corresponding to an "infection or assimilation effect") we will observe a monotonic relaxation towards the equilibrium which is – for otherwise equivalent conditions concerning B and R – characterized by equipartition. Yet introduction of a memory changes the picture. If we assume that B converts to R only after having met B n-times (see Suppl. 4), damped oscillations towards the equilibrium distribution occur, with the damping being the less pronounced the greater n. As shown in the Supplementary Information and alluded to in the beginning, a slightly modified version is arrived at when the changed is induced by a collision with a third party (the concentration of which can be kept constant), named a converter (rather than with another colored particle). This version has the advantage that it can be easily brought into connection with the thermodynamic approach.

With these considerations of initial conditions and more realistic behavioral aspects we did not intend to distract from the major point namely highlighting the general tendency of many sociological systems to oscillate even under simple mechanistic conditions, rather it was meant to show the aptitude of the simple model for refinement.

7. Conclusions

In short, we presented a simple but realistic toy-model of over-/underestimation cycles based on the accumulated discrepancy between perceived value and real value, or in more general terms between seeming and being. Moreover, the model will be of use (with appropriate renaming of the variables and parameters) whenever effects of deviations from the true value (in the most general sense of the word) are of sociological influence. What makes sociological systems easily prone to oscillate (even for simple mechanisms) is the significance of memory and persistence that endows the system with the necessary inertia. The model is purposefully strongly simplified and may – given the complexity of real phenomena – appear as naïve, but it is believed to hit basic points and hence to enable sharper definitions of otherwise extremely loosely defined terms. It is not in conflict with the fact that sociological events can be mechanistically complex and highly non-linear, it rather can serve as a platform allowing for specific extensions. The model stresses the usefulness of sociological equivalent circuits and also exhibits the flexibility to incorporate sociological prehistory as well as learning effects leading to a more realistic description.

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Supplementary 1: Cyclic Processes: Kinetics and Trajectories

We refer to linear first order differential equations of the type $\frac{dx_1}{dt} = F(x_1, x_2 ...)$, whereby the variables x_1, x_2 ... depend on time. Such equations are well-known from chemical (reaction rate equations) and physical kinetics (master equations) and express the fact that the rates are determined by encounter and transition probabilities. Also, from a mathematical standpoint, they are more general than it appears, as any linear ordinary differential equation can be transformed into a set of first order differential equations.

The condition $F(\hat{x}_1, \hat{x}_2 ...) = 0$ yields the stationary points $\hat{x}_1$, $\hat{x}_2$ etc. The stability analysis is based on investigating the response to a small displacement from the stationary points. If the response drives the system back to the stationary values, the stationary point is stable (negative feedback); if the response drives the system away from the stationary values, the stationary point is unstable. For the consideration of oscillations, two patterns are particularly interesting. The first refers to ellipses around the stationary point corresponding to undamped harmonic oscillations around the stationary value. An example is provided by

$dx_1/dt = ax_2$ and $dx_2/dt = -bx_1$. (Eq S1, S2)

Eliminating dt yields dx_1/dx_2; integration leads to $x_1^2 b + x_2^2 a = \text{const.}$ which characterizes an ellipse. (Eqs S1, S2) refer to a truncated Lotka-Volterra scheme without mixed terms ($x_1 x_2$). The trajectories of the full system are no longer ellipses but still closed loops (LENTZ 1993). The corresponding stationary values are stable, but not asymptotically stable. These equations are also referred to as structurally unstable, since admixing smallest contributions of x_1 to the rhs of the Eq S1 and x_2 to the rhs of Eq S2 destroys the closed loops.

The other type of interest is the instability of spiral type. In a self-excited scenario, a displacement leads to an outward spiral (away from the stationary point) corresponding to motions with ever increasing amplitudes. In practical systems this increase is limited, and a limit cycle is obtained. A limit cycle can also be reached via an inward spiral meaning that the amplitudes of damped oscillations decrease to a constant value.

The reader should note: Phenomena such as limit cycles, bifurcations (emergence of new patterns, innovations) and chaotic phenomena (fractalization) follow only if non-linear differential equations are referred to. More details can be found in FEISTEL and EBELING (2011), EBELING (1976), MINORSKY (1962), KOTOMIN and KUZOVKOV (1996), LENTZ (1993).

Supplementary 2: Cyclic Processes: Irreversible Thermodynamics

Let us consider a defined open system. The entropy change can be split into a contribution stemming from the environment and a contribution stemming from internal entropy production. The second law demands a positive entropy production for any process and a zero value for equilibrium. The process under concern is characterized by a rate (flux) $\mathcal{R}$ and a driving force $\mathcal{A}$ (affinity), leading to an entropy production given by $\mathcal{RA}$. In equilibrium both $\mathcal{R}$ and $\mathcal{A}$ are zero and so is the entropy production. The entropy production is obviously also zero for a frozen-in situation as $\mathcal{R} = 0$ whilst $\mathcal{A} \neq 0$.

PRIGOGINE showed that in the regime close to equilibrium where $\mathcal{R} \propto \mathcal{A}$, stable stationary values can occur, the entropy production of which is non-zero but at minimum (NICOLIS and PRIGOGINE 1985, GLANSDORFF and PRIGOGINE 1971). A small displacement leads back to the stationary value excluding the possibility of oscillations. Far from equilibrium however, such oscillations are possible as there the above statements are only valid for that part of the entropy production that stems from the variation of forces (GLANSDORFF and PRIGOGINE 1971). It must be noted that the range of validity of the Glansdorff-Prigogine criterion is still under debate (FEISTEL and EBELING 2011). (It is important to note that the criterion requires temperature and chemical potentials to be time-invariant in any point of the system's boundary. Many contradictions in literature are due to not paying attention to these boundary conditions).

In the non-linear regime it is now possible that displacements lead to leaving the stationary state to the favour of another one. These considerations are important for evolution in biology (dissipative structures, EIGEN 1971) and economy (innovations, KONDRATJEW 1926).

For more detailed literature see GLANSDORFF and PRIGOGINE (1971), NICOLIS and PRIGOGINE (1985), FEISTEL and EBELING (2011), MAIER (2023), EIGEN (1971).

Supplementary 3: Cycles Based on Time-delay

If $a(t) \propto \Delta(t \pm \tau)$ but $\dot{a}(t) \propto \Delta(t)$, the natural solution is given by sin-functions, as the derivative of a sin-function is a cos-function which shows a phase-shift to the sin-function of $\pi/2 = \omega\tau$.

Supplementary 4: The Fashion Problem

This brief consideration deals with intrinsically caused fashion cycles. (Extrinsic effects such as periodic supply of fashion articles by fashion industry to avoid saturation are not taken account of even though important in praxi, such forced oscillations are rather trivial in the present context.) We assume two colors blue (B) and red (R). There are two effects of interest. One is the "exclusivity effect" consisting in that B on meeting B will change to R (and vice versa). The other is the "assimilation, imitation or infection effect" consisting in B on meeting R will change to R (and vice versa). We concentrate on the first effect and set all the kinetic constants to unity.

Hence, for the exclusivity effect B + B → R + R and R + R → B + B are the important reactions which for simplicity are supposed to run at the same rates. If we assume simple reaction kinetics both b and r (small characters denote the respective concentrations) will monotonically relax from the initial value to the equilibrium value $\hat{b} = \hat{r} = 1/2$.

If we now introduce a memory such that B changes to R if only B has encountered B n-times and vice versa for R; then we need to distinguish between B's that have had such an encounter for the first, second, … n'th time. If, e.g., n=2 we need to consider the events

$$\begin{aligned}
B_0 + B_0 &\rightarrow B_1 + B_1 \\
B_0 + B_1 &\rightarrow B_1 + B_2 \\
B_0 + B_2 &\rightarrow B_1 + R_0 \\
B_1 + B_1 &\rightarrow B_2 + B_2 \\
B_1 + B_2 &\rightarrow B_2 + R_0 \\
B_2 + B_2 &\rightarrow R_0 + R_0
\end{aligned}$$

and the same for R.

Note that these equations do not satisfy microscopic reversibility and should not be used to describe chemical reactions. (In principle the rate constants should depend on the index of the reaction partners. This variation is neglected here.) Figure S 4 gives solutions for various n showing that the introduction of a memory leads to damped oscillations (initially 1000 blue and 9000 red particles).

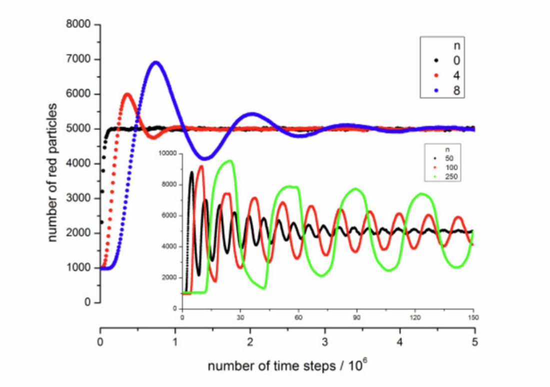


Fig. S 4

Certainly, the "infection effect", viz. the tendency of B to change to R on meeting R could be easily implemented as well.

The introduction of different species corresponds to a discrete transition from spontaneous to accumulative behavior. Hence damped oscillations occur, the behavior of which depends on the strength of the memory and reaction readiness (A_1,...,A_n; n is related with memory and α).

A simplified version of the above is obtained if we demand B (or R) to react on meeting a "converter" Z, the concentration of which we can keep constant (see Figure 1, main text). Then, e.g., for n = 2

$$\begin{aligned} &\mathrm{B_0 + Z \rightarrow B_1 + Z} \\ &\mathrm{B_1 + Z \rightarrow B_2 + Z} \\ &\mathrm{B_2 + Z \rightarrow R_0 + Z} \end{aligned}$$

and the same for R.

Even though these equations are now (as the concentration of Z is const) of first order, the general picture is not very different. The reaction without memory $\mathrm{B + Z \rightleftarrows R + Z}$ is describable by an exponential function that satisfies the rate law

$$\text{rate} \propto \text{b-}\hat{\text{b}} = \text{-}\left(\text{r}-\hat{\text{r}}\right).$$

As described in the main text; in order to treat more general situations, we proceed to a description that is not necessarily based on collision events but includes memory by demanding

$$\text{rate} \propto \int\left(\text{b-}\hat{\text{b}}\right)dt.$$

Unlike in the master example, this problem (given $\hat{\text{b}}$) only requires a single sociological parameter, viz. the proportionality factor in this equation. As in the master example it refers to the reaction readiness of the system.

Supplementary 5: Harmonic Oscillations

$-\ddot{y} = Ay + B$ has the solution $y = \frac{p}{A}\sin\omega t - \frac{B}{A}$ if the phase shift is assumed to be zero with the frequency $\omega = \sqrt{A}$ and oscillations around $-\frac{B}{A}$.

Using $\varepsilon = \partial E / \partial e$, $\eta = \partial a / \partial E$, we consider 2 examples:

1) Example $e = \hat{e}$

$$-\ddot{a} = \alpha(E - \hat{e}) = -\eta\ddot{E}$$

$$-\ddot{\boldsymbol{E}} = \frac{\boldsymbol{\alpha}}{\boldsymbol{\eta}}\boldsymbol{E} - \frac{\boldsymbol{\alpha}}{\boldsymbol{\eta}}\hat{e}$$

Solution for E follows as above ($E \equiv y$) with $A = \alpha/\eta$ and $B = -\alpha\hat{e}/\eta$. Hence E oscillates around $\frac{\alpha}{\eta}\hat{e}\frac{\eta}{\alpha} = \hat{e}$ with the frequency $\omega = \sqrt{\frac{\alpha}{\eta}}$.

2) Example $\boldsymbol{E - e = \delta E - \delta e + \underbrace{(\hat{E} - \hat{e})}_{\hat{\Delta}}}$

$$\ddot{a} = -\alpha(\delta E - \delta e + \hat{\Delta}) = (\delta a)^{\cdot\cdot} = \eta(\delta E)^{\cdot\cdot}$$

$$-(\boldsymbol{\delta E})^{\cdot\cdot} = \frac{\boldsymbol{\alpha}}{\boldsymbol{\eta}}\frac{\boldsymbol{\varepsilon}-1}{\boldsymbol{\varepsilon}}\boldsymbol{\delta E} + \frac{\boldsymbol{\alpha}}{\boldsymbol{\eta}}\hat{\Delta}$$

Solution for δE ($\equiv y$) follows as above with $A = \frac{\alpha}{\eta}\frac{\varepsilon-1}{\varepsilon}$ and $B = \frac{\alpha}{\eta}\hat{\Delta}$. Hence δE oscillates around $-\frac{\alpha}{\eta}\hat{\Delta}\frac{\eta}{\alpha}\frac{\varepsilon}{\varepsilon-1} = -\hat{\Delta}\frac{\varepsilon}{\varepsilon-1}$ with a frequency $\omega = \sqrt{\frac{\alpha}{\eta}\frac{\varepsilon-1}{\varepsilon}}$, i.e. E oscillates around $-\hat{\Delta}\frac{\varepsilon}{\varepsilon-1} + \hat{E} = \frac{1}{\varepsilon-1}(\varepsilon\hat{e} - \hat{E})$;

δe oscillates around $-\hat{\Delta}\frac{1}{\varepsilon-1}$, i.e. e oscillates around $-(\hat{E}-\hat{e})\frac{1}{\varepsilon-1} + \hat{e} = \frac{1}{\varepsilon-1}(\varepsilon\hat{e} - \hat{E})$;

δa oscillates around $-\eta\frac{\varepsilon}{\varepsilon-1}\hat{\Delta}$, i.e. a oscillates around $-\frac{\eta\varepsilon}{\varepsilon-1}\hat{\Delta} + \hat{a}$.

– If $\varepsilon \to 1$ then the term $\frac{\varepsilon}{\varepsilon-1}\hat{\Delta}$ is finite only if $\hat{\Delta} = 0$ ("expectation equilibrium").
– If $\varepsilon \to \infty$ then, according to the definition of ε, $e = \hat{e}$ with oscillations around 0 (for δE) and around $\hat{e}$ (for E), respectively (see Example 1).
– If $\hat{e} = \hat{E}$, i.e. $\hat{\Delta} = 0$, then oscillations are around 0 (for δE) and around $\hat{e} = \hat{E}$ (for E), respectively.

Supplementary 6

Solutions for $\ddot{a}(\tau) = a^{n}(\tau) - a^{m}(\tau)$ with selected n, m:

a) The differential equation $\ddot{a}(\tau) = a^{1/2}(\tau) - a(\tau)$ has the implicit solution

$$\tau = -2\,ArcTan\left[\frac{\left(-2+3\sqrt{a}\right)\sqrt{4a^{3/2}-3a^{2}}}{\sqrt{3\left(-4+3\sqrt{a}\right)a}}\right]$$

or explicitly for trivial boundary conditions

$$a(\tau) = a_1(\tau)\ @\ a_2(\tau)$$

$$a_1 = \frac{4\left(1+3Tan\left[\frac{t}{2}\right]^2+2Tan\left[\frac{t}{2}\right]^4 - 2\sqrt{Tan\left[\frac{t}{2}\right]^2+3Tan\left[\frac{t}{2}\right]^4+3Tan\left[\frac{t}{2}\right]^6+Tan\left[\frac{t}{2}\right]^8}\right)}{9\left(1+2Tan\left[\frac{t}{2}\right]^2+Tan\left[\frac{t}{2}\right]^4\right)}$$

$$a_2 = \frac{4\left(1+3Tan\left[\frac{t}{2}\right]^2+2Tan\left[\frac{t}{2}\right]^4 + 2\sqrt{Tan\left[\frac{t}{2}\right]^2+3Tan\left[\frac{t}{2}\right]^4+3Tan\left[\frac{t}{2}\right]^6+Tan\left[\frac{t}{2}\right]^8}\right)}{9\left(1+2Tan\left[\frac{t}{2}\right]^2+Tan\left[\frac{t}{2}\right]^4\right)}$$

as sketched in Fig. S 6A.

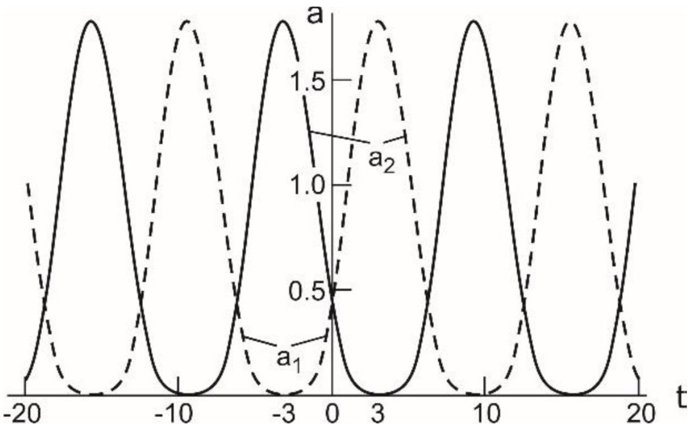


Fig. S 6A

The intercepts with the τ -axis are odd multiples of π; the intercept with the a-axis is 4/9. The inflection point is at a = 1; the maximum is 16/9. The oscillating functions are composites of a_1 and a_2.

b) The solution for $\ddot{a}(\tau) = a^2(\tau) - a(\tau)$ is for trivial boundary conditions

$$a(\tau) = \frac{3}{2}\left(1 + \left(Tan\left(\frac{\tau}{2}\right)\right)^2\right).$$

The graph is sketched in Fig. S 6B.

$$plot\left(\frac{3}{2}\cdot\left(1+\left(\tan\left(\frac{\text{tau}}{2}\right)\right)^2\right), \text{tau}=-15..15, -10..50\right);$$

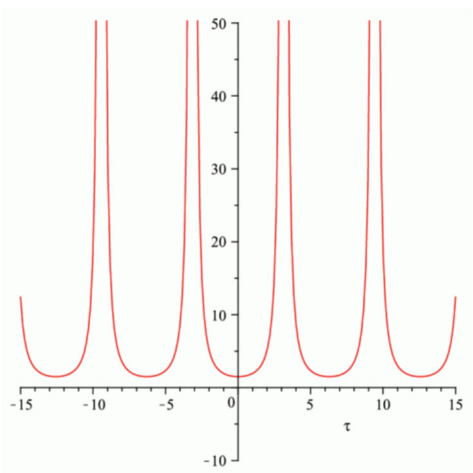


Fig. S 6B

c) The differential equation

$$\ddot{a}(\tau) = a^{3/2}(\tau) - a(\tau)$$

delivers a periodical but non-oscillating function

$$a(\tau) = \frac{25}{16}\left[1 + \tan\left[\frac{\tau}{4}\right]^2 + \tan\left[\frac{\tau}{4}\right]^4\right] .$$

The a (τ) curves are given in Fig. S 6C.

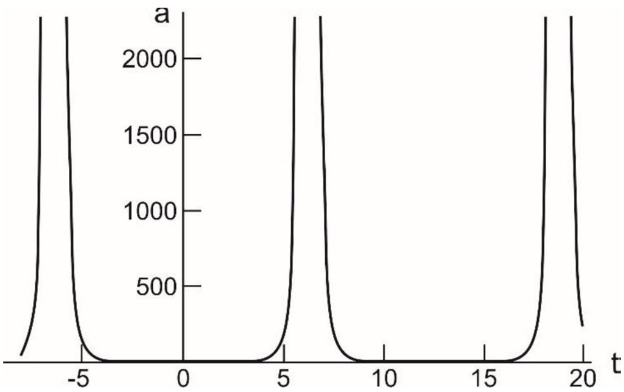


Fig. S 6C

d) It is interesting to compare the solutions for $\ddot{a}(\tau)=a^n(\tau)-a^m(\tau)$ with those for the spontaneous reaction $\dot{a}(\tau)=a^n(\tau)-a^m(\tau)$. Let us first consider $\dot{a}(\tau)=a(\tau)-a^2(\tau)$.

The solution for a (0) ≠ 1 is

$$a(\tau) = \frac{\frac{a(0)}{1-a(0)}\exp\tau}{1+\frac{a(0)}{1-a(0)}\exp\tau} \quad .$$

For $a < 1$ the first term predominates and a increases. For $a > 1$ the second term predominates and a decreases. Yet, unlike the accumulative response, oscillations do not occur, rather the value $a = 1$ forms a boundary for the solutions. If a ($\tau = 0$) ≡ a (0) > 1, the a-function stays always greater than unity, while it stays below unity if $a(0)<1$. See Fig. S 6D.

A similar compartmentation occurs for the solution of $\dot{a}(\tau) = a^2(\tau) - a(\tau)$ which reads for $a\,(0) \neq 1$

$$a(\tau) = -\frac{\dfrac{a(0)}{a(0)-1}}{\exp\tau - \dfrac{a(0)}{a(0)-1}}$$

$$plot\left(\left\{\frac{4}{3}\cdot\frac{\exp(\mathrm{tau})}{1+\left(\frac{4}{3}\right)\cdot\exp(\mathrm{tau})}, -\frac{\exp(\mathrm{tau})}{1-\exp(\mathrm{tau})}\right\}, \mathrm{tau}=-5..5, -5..5\right);$$

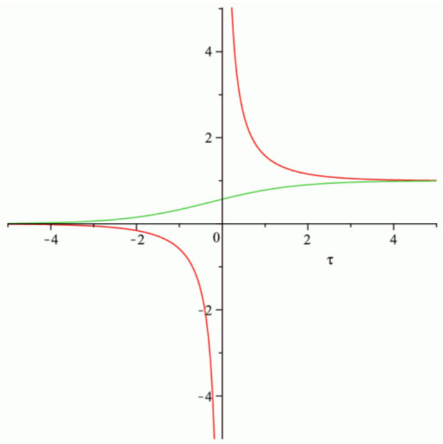


Fig. S 6D

and is displayed in Fig. S 6E, which may be compared with Fig. 2B. Such a compartmentation is dissolved for an accumulative response.

$$plot\left(\left\{-\frac{4}{3}\cdot\left(\exp(\mathrm{tau})-\frac{4}{3}\right)^{-1}, (\exp(\mathrm{tau})+1)^{-1}\right\}, \mathrm{tau}=-5..5, -5..5\right);$$

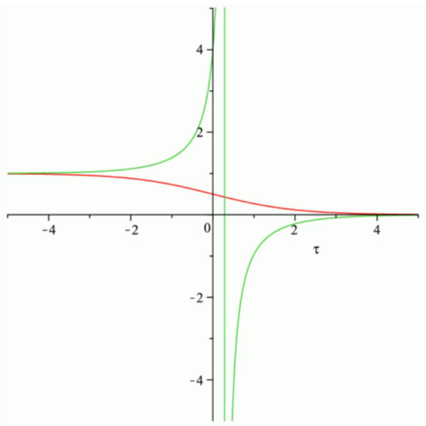


Fig. S 6E

Supplementary 7: Taboo-phase and Breakdown

Let us for simplicity assume a direct, spontaneous reaction (rather than an accumulative one), which enables us to adopt the simple formalism of chemical kinetics. The taboo-phase (or phase of directorship) is determined by the fact that any unwanted action (activity) must rapidly decay (large reaction constant $\vec{k}_D$) according to

$$\mathrm{A} \xrightarrow{\vec{k}_D} \mathrm{Nil}. \tag{1}$$

Nonetheless there is also a growth mechanism that is enabled wherever a precursor A* is met (the activity of A* is an activity in a critical pre-stage)

$$\mathrm{A} + \mathrm{A}^* \xrightarrow{\vec{k}_G} 2\mathrm{A}. \tag{2}$$

The overall rate is $[\mathrm{A}]\left(\vec{k}_G\left[\mathrm{A}^*\right]-\vec{k}_D\right)$. At a high enough number density of A* (i.e. of [A*]), namely for $\vec{k}_G\left[\mathrm{A}^*\right] > \vec{k}_D$ channel 2 dominates: A grows, finally leading to a breakdown of the taboo-phase. The fate of it is determined by the ratio $\frac{\vec{k}_G\left[\mathrm{A}^*\right]}{\vec{k}_D}$. The situation becomes very interesting, if several activities compete with each other and compete about an exhaustible number of A*. This leads automatically to a selection problem in which the growth (survival) depends on other activities. Incorporation of memory effects to the treatment given in (EIGEN 1971) should lead to analogous solutions for sociological problems.

Supplementary 8: Solutions Including Pre-phases

Supplementary 8a

$$dgl := diff(y(x), x, x) + 100 \cdot \left(1 - \exp\left(-\frac{x}{10}\right)\right) \cdot y(x) = 0;$$

$$\frac{d^2}{dx^2} y(x) + 100\left(1 - e^{-\frac{1}{10}x}\right) y(x) = 0$$

$$dsolve(\{dgl, y(0) = -1000, D(y)(0) = 0\}, y(x));$$

$$y(x) = \Big(1000\,(-\mathrm{BesselK}(1 + 200\,I, 200) + I\,\mathrm{BesselK}(200\,I, 200))\,\mathrm{BesselI}\left(200\,I, 200\,e^{-\frac{1}{20}x}\right)\Big) \Big/ (\mathrm{BesselK}(200\,I, 200)\,\mathrm{BesselI}(1 + 200\,I, 200) + \mathrm{BesselK}(1 + 200\,I, 200)\,\mathrm{BesselI}(200\,I, 200)) - \Big(1000\,(\mathrm{BesselI}(1 + 200\,I, 200) + I\,\mathrm{BesselI}(200\,I, 200))\,\mathrm{BesselK}\left(200\,I, 200\,e^{-\frac{1}{20}x}\right)\Big) \Big/ (\mathrm{BesselK}(200\,I, 200)\,\mathrm{BesselI}(1 + 200\,I, 200) + \mathrm{BesselK}(1 + 200\,I, 200)\,\mathrm{BesselI}(200\,I, 200))$$

$$funcy := unapply(rhs(dsolve(\{dgl, y(0) = -1000, D(y)(0) = 0\}, y(x))), x);$$

$$x \rightarrow \Big(1000\,(-\mathrm{BesselK}(1 + 200\,I, 200) + I\,\mathrm{BesselK}(200\,I, 200))\,\mathrm{BesselI}\left(200\,I, 200\,e^{-\frac{1}{20}x}\right)\Big) \Big/ (\mathrm{BesselK}(200\,I, 200)\,\mathrm{BesselI}(1 + 200\,I, 200) + \mathrm{BesselK}(1 + 200\,I, 200)\,\mathrm{BesselI}(200\,I, 200)) - \Big(1000\,(\mathrm{BesselI}(1 + 200\,I, 200) + I\,\mathrm{BesselI}(200\,I, 200))\,\mathrm{BesselK}\left(200\,I, 200\,e^{-\frac{1}{20}x}\right)\Big) \Big/ (\mathrm{BesselK}(200\,I, 200)\,\mathrm{BesselI}(1 + 200\,I, 200) + \mathrm{BesselK}(1 + 200\,I, 200)\,\mathrm{BesselI}(200\,I, 200))$$

$$plot(funcy(x), x = -0.5..5);$$

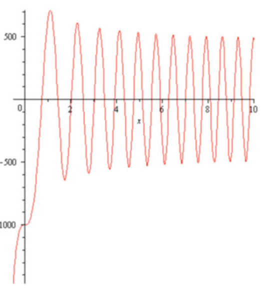


Fig. S 8a

Supplementary 8b

dgl $:= \mathit{diff}(y(x), x, x) + \left(2 - 1.99 \cdot \exp\left(-\frac{x}{5}\right)\right)^{-1} \cdot y(x) = 0;$

$$\frac{d^2}{dx^2} y(x) + \frac{y(x)}{2 - 1.99 e^{-\frac{1}{5}x}} = 0 \qquad \textbf{(1)}$$

dsolve$(\{dgl, y(0) = 10, \mathrm{D}(y)(0) = 0\}, y(x));$

$$y(x) = -\Big(10\Big(102000\,\mathrm{I\,hypergeom}\Big(\Big[1+\frac{5}{2}\,\mathrm{I}\sqrt{2}, 1+\frac{5}{2}\,\mathrm{I}\sqrt{2}\Big], \Big[1+5\,\mathrm{I}\sqrt{2}\Big], \frac{199}{200}\Big)\sqrt{2} - 8119200\,\mathrm{hypergeom}\Big(\Big[1 \qquad \textbf{(2)}$$

$$+\frac{5}{2}\,\mathrm{I}\sqrt{2}, 1+\frac{5}{2}\,\mathrm{I}\sqrt{2}\Big], \Big[1+5\,\mathrm{I}\sqrt{2}\Big], \frac{199}{200}\Big) + 30646\,\mathrm{hypergeom}\Big(\Big[2+\frac{5}{2}\,\mathrm{I}\sqrt{2}, 2+\frac{5}{2}\,\mathrm{I}\sqrt{2}\Big], \Big[2+5\,\mathrm{I}\sqrt{2}\Big],$$

$$\frac{199}{200}\Big) + 49750\,\mathrm{I\,hypergeom}\Big(\Big[2+\frac{5}{2}\,\mathrm{I}\sqrt{2}, 2+\frac{5}{2}\,\mathrm{I}\sqrt{2}\Big], \Big[2+5\,\mathrm{I}\sqrt{2}\Big], \frac{199}{200}\Big)\sqrt{2}\Big)\Big(-200\,e^{\frac{1}{2}\mathrm{I}\sqrt{2}x}$$

$$+199\,e^{\frac{1}{2}\mathrm{I}\sqrt{2}x - \frac{1}{5}x}\Big)\,\mathrm{hypergeom}\Big(\Big[1-\frac{5}{2}\,\mathrm{I}\sqrt{2}, 1-\frac{5}{2}\,\mathrm{I}\sqrt{2}\Big], \Big[1-5\,\mathrm{I}\sqrt{2}\Big], \frac{199}{200}\,e^{-\frac{1}{5}x}\Big)\Big) \Big/$$

$$\Big(204000\,\mathrm{I\,hypergeom}\Big(\Big[1+\frac{5}{2}\,\mathrm{I}\sqrt{2}, 1+\frac{5}{2}\,\mathrm{I}\sqrt{2}\Big], \Big[1+5\,\mathrm{I}\sqrt{2}\Big], \frac{199}{200}\Big)\sqrt{2}\,\mathrm{hypergeom}\Big(\Big[1-\frac{5}{2}\,\mathrm{I}\sqrt{2}, 1$$

$$-\frac{5}{2}\,\mathrm{I}\sqrt{2}\Big], \Big[1-5\,\mathrm{I}\sqrt{2}\Big], \frac{199}{200}\Big) - 30646\,\mathrm{hypergeom}\Big(\Big[1+\frac{5}{2}\,\mathrm{I}\sqrt{2}, 1+\frac{5}{2}\,\mathrm{I}\sqrt{2}\Big], \Big[1+5\,\mathrm{I}\sqrt{2}\Big],$$

$$\frac{199}{200}\Big)\,\mathrm{hypergeom}\Big(\Big[2-\frac{5}{2}\,\mathrm{I}\sqrt{2}, 2-\frac{5}{2}\,\mathrm{I}\sqrt{2}\Big], \Big[2-5\,\mathrm{I}\sqrt{2}\Big], \frac{199}{200}\Big) + 49750\,\mathrm{I\,hypergeom}\Big(\Big[1+\frac{5}{2}\,\mathrm{I}\sqrt{2}, 1$$

$$+\frac{5}{2}\,\mathrm{I}\sqrt{2}\Big], \Big[1+5\,\mathrm{I}\sqrt{2}\Big], \frac{199}{200}\Big)\,\mathrm{hypergeom}\Big(\Big[2-\frac{5}{2}\,\mathrm{I}\sqrt{2}, 2-\frac{5}{2}\,\mathrm{I}\sqrt{2}\Big], \Big[2-5\,\mathrm{I}\sqrt{2}\Big], \frac{199}{200}\Big)\sqrt{2}$$

$$+30646\,\mathrm{hypergeom}\Big(\Big[2+\frac{5}{2}\,\mathrm{I}\sqrt{2}, 2+\frac{5}{2}\,\mathrm{I}\sqrt{2}\Big], \Big[2+5\,\mathrm{I}\sqrt{2}\Big], \frac{199}{200}\Big)\,\mathrm{hypergeom}\Big(\Big[1-\frac{5}{2}\,\mathrm{I}\sqrt{2}, 1-\frac{5}{2}\,\mathrm{I}\sqrt{2}\Big], \Big[1$$

$$-5\,\mathrm{I}\sqrt{2}\Big], \frac{199}{200}\Big) + 49750\,\mathrm{I\,hypergeom}\Big(\Big[2+\frac{5}{2}\,\mathrm{I}\sqrt{2}, 2+\frac{5}{2}\,\mathrm{I}\sqrt{2}\Big], \Big[2+5\,\mathrm{I}\sqrt{2}\Big], \frac{199}{200}\Big)\sqrt{2}\,\mathrm{hypergeom}\Big(\Big[1$$

$$-\frac{5}{2}\,\mathrm{I}\sqrt{2}, 1-\frac{5}{2}\,\mathrm{I}\sqrt{2}\Big], \Big[1-5\,\mathrm{I}\sqrt{2}\Big], \frac{199}{200}\Big)\Big) - \Big(10\Big(102000\,\mathrm{I\,hypergeom}\Big(\Big[1-\frac{5}{2}\,\mathrm{I}\sqrt{2}, 1-\frac{5}{2}\,\mathrm{I}\sqrt{2}\Big],$$

$$\Big[1-5\,\mathrm{I}\sqrt{2}\Big], \frac{199}{200}\Big)\sqrt{2} + 8119200\,\mathrm{hypergeom}\Big(\Big[1-\frac{5}{2}\,\mathrm{I}\sqrt{2}, 1-\frac{5}{2}\,\mathrm{I}\sqrt{2}\Big], \Big[1-5\,\mathrm{I}\sqrt{2}\Big], \frac{199}{200}\Big)$$

$$-30646\,\mathrm{hypergeom}\Big(\Big[2-\frac{5}{2}\,\mathrm{I}\sqrt{2}, 2-\frac{5}{2}\,\mathrm{I}\sqrt{2}\Big], \Big[2-5\,\mathrm{I}\sqrt{2}\Big], \frac{199}{200}\Big) + 49750\,\mathrm{I\,hypergeom}\Big(\Big[2-\frac{5}{2}\,\mathrm{I}\sqrt{2}, 2$$

$$-\frac{5}{2}\,\mathrm{I}\sqrt{2}\Big], \Big[2-5\,\mathrm{I}\sqrt{2}\Big], \frac{199}{200}\Big)\sqrt{2}\Big)\Big(-200\,e^{-\frac{1}{2}\mathrm{I}\sqrt{2}x} + 199\,e^{-\frac{1}{2}\mathrm{I}\sqrt{2}x - \frac{1}{5}x}\Big)\,\mathrm{hypergeom}\Big(\Big[1+\frac{5}{2}\,\mathrm{I}\sqrt{2}, 1$$

$$+\frac{5}{2}\,\mathrm{I}\sqrt{2}\Big], \Big[1+5\,\mathrm{I}\sqrt{2}\Big], \frac{199}{200}\,e^{-\frac{1}{5}x}\Big)\Big)\Big/\Big(204000\,\mathrm{I\,hypergeom}\Big(\Big[1+\frac{5}{2}\,\mathrm{I}\sqrt{2}, 1+\frac{5}{2}\,\mathrm{I}\sqrt{2}\Big], \Big[1+5\,\mathrm{I}\sqrt{2}\Big],$$

$$\frac{199}{200}\Big)\sqrt{2}\,\mathrm{hypergeom}\Big(\Big[1-\frac{5}{2}\,\mathrm{I}\sqrt{2}, 1-\frac{5}{2}\,\mathrm{I}\sqrt{2}\Big], \Big[1-5\,\mathrm{I}\sqrt{2}\Big], \frac{199}{200}\Big) - 30646\,\mathrm{hypergeom}\Big(\Big[1+\frac{5}{2}\,\mathrm{I}\sqrt{2}, 1$$

$$+\frac{5}{2}\,\mathrm{I}\sqrt{2}\Big], \Big[1+5\,\mathrm{I}\sqrt{2}\Big], \frac{199}{200}\Big)\,\mathrm{hypergeom}\Big(\Big[2-\frac{5}{2}\,\mathrm{I}\sqrt{2}, 2-\frac{5}{2}\,\mathrm{I}\sqrt{2}\Big], \Big[2-5\,\mathrm{I}\sqrt{2}\Big], \frac{199}{200}\Big)$$

$$+49750\,\mathrm{I\,hypergeom}\Big(\Big[1+\frac{5}{2}\,\mathrm{I}\sqrt{2}, 1+\frac{5}{2}\,\mathrm{I}\sqrt{2}\Big], \Big[1+5\,\mathrm{I}\sqrt{2}\Big], \frac{199}{200}\Big)\,\mathrm{hypergeom}\Big(\Big[2-\frac{5}{2}\,\mathrm{I}\sqrt{2}, 2-\frac{5}{2}\,\mathrm{I}\sqrt{2}\Big],$$

$$\Big[2-5\,\mathrm{I}\sqrt{2}\Big], \frac{199}{200}\Big)\sqrt{2} + 30646\,\mathrm{hypergeom}\Big(\Big[2+\frac{5}{2}\,\mathrm{I}\sqrt{2}, 2+\frac{5}{2}\,\mathrm{I}\sqrt{2}\Big], \Big[2+5\,\mathrm{I}\sqrt{2}\Big], \frac{199}{200}\Big)\,\mathrm{hypergeom}\Big(\Big[1$$

$$-\frac{5}{2}\,\mathrm{I}\sqrt{2}, 1-\frac{5}{2}\,\mathrm{I}\sqrt{2}\Big], \Big[1-5\,\mathrm{I}\sqrt{2}\Big], \frac{199}{200}\Big) + 49750\,\mathrm{I\,hypergeom}\Big(\Big[2+\frac{5}{2}\,\mathrm{I}\sqrt{2}, 2+\frac{5}{2}\,\mathrm{I}\sqrt{2}\Big], \Big[2+5\,\mathrm{I}\sqrt{2}\Big],$$

$$\frac{199}{200}\Big)\sqrt{2}\,\mathrm{hypergeom}\Big(\Big[1-\frac{5}{2}\,\mathrm{I}\sqrt{2}, 1-\frac{5}{2}\,\mathrm{I}\sqrt{2}\Big], \Big[1-5\,\mathrm{I}\sqrt{2}\Big], \frac{199}{200}\Big)\Big)$$

funcy := *unapply*(*rhs*(*dsolve*({*dgl*, *y*(0) = 10, D(*y*)(0) = 0}, *y*(*x*))), *x*);

$$
\begin{aligned}
& x \to -\Big(10\,\Big(102000\,\mathrm{I}\,\mathrm{hypergeom}\Big(\Big[1+\tfrac{5}{2}\,\mathrm{I}\sqrt{2},\,1+\tfrac{5}{2}\,\mathrm{I}\sqrt{2}\Big],\,\Big[1+5\,\mathrm{I}\sqrt{2}\Big],\,\tfrac{199}{200}\Big)\sqrt{2}-8119200\,\mathrm{hypergeom}\Big(\Big[1 \qquad (3)\\
& +\tfrac{5}{2}\,\mathrm{I}\sqrt{2},\,1+\tfrac{5}{2}\,\mathrm{I}\sqrt{2}\Big],\,\Big[1+5\,\mathrm{I}\sqrt{2}\Big],\,\tfrac{199}{200}\Big)+30646\,\mathrm{hypergeom}\Big(\Big[2+\tfrac{5}{2}\,\mathrm{I}\sqrt{2},\,2+\tfrac{5}{2}\,\mathrm{I}\sqrt{2}\Big],\,\Big[2+5\,\mathrm{I}\sqrt{2}\Big],\\
& \tfrac{199}{200}\Big)+49750\,\mathrm{I}\,\mathrm{hypergeom}\Big(\Big[2+\tfrac{5}{2}\,\mathrm{I}\sqrt{2},\,2+\tfrac{5}{2}\,\mathrm{I}\sqrt{2}\Big],\,\Big[2+5\,\mathrm{I}\sqrt{2}\Big],\,\tfrac{199}{200}\Big)\sqrt{2}\Big)\Big(-200\,\mathrm{e}^{\frac{1}{2}\mathrm{I}\sqrt{2}\,x}\\
& +199\,\mathrm{e}^{\frac{1}{2}\mathrm{I}\sqrt{2}\,x-\frac{1}{5}x}\Big)\,\mathrm{hypergeom}\Big(\Big[1-\tfrac{5}{2}\,\mathrm{I}\sqrt{2},\,1-\tfrac{5}{2}\,\mathrm{I}\sqrt{2}\Big],\,\Big[1-5\,\mathrm{I}\sqrt{2}\Big],\,\tfrac{199}{200}\,\mathrm{e}^{-\frac{1}{5}x}\Big)\Big)\Big/\\
& \Big(204000\,\mathrm{I}\,\mathrm{hypergeom}\Big(\Big[1+\tfrac{5}{2}\,\mathrm{I}\sqrt{2},\,1+\tfrac{5}{2}\,\mathrm{I}\sqrt{2}\Big],\,\Big[1+5\,\mathrm{I}\sqrt{2}\Big],\,\tfrac{199}{200}\Big)\sqrt{2}\,\mathrm{hypergeom}\Big(\Big[1-\tfrac{5}{2}\,\mathrm{I}\sqrt{2},\,1\\
& -\tfrac{5}{2}\,\mathrm{I}\sqrt{2}\Big],\,\Big[1-5\,\mathrm{I}\sqrt{2}\Big],\,\tfrac{199}{200}\Big)-30646\,\mathrm{hypergeom}\Big(\Big[1+\tfrac{5}{2}\,\mathrm{I}\sqrt{2},\,1+\tfrac{5}{2}\,\mathrm{I}\sqrt{2}\Big],\,\Big[1+5\,\mathrm{I}\sqrt{2}\Big],\\
& \tfrac{199}{200}\Big)\,\mathrm{hypergeom}\Big(\Big[2-\tfrac{5}{2}\,\mathrm{I}\sqrt{2},\,2-\tfrac{5}{2}\,\mathrm{I}\sqrt{2}\Big],\,\Big[2-5\,\mathrm{I}\sqrt{2}\Big],\,\tfrac{199}{200}\Big)+49750\,\mathrm{I}\,\mathrm{hypergeom}\Big(\Big[1+\tfrac{5}{2}\,\mathrm{I}\sqrt{2},\,1\\
& +\tfrac{5}{2}\,\mathrm{I}\sqrt{2}\Big],\,\Big[1+5\,\mathrm{I}\sqrt{2}\Big],\,\tfrac{199}{200}\Big)\,\mathrm{hypergeom}\Big(\Big[2-\tfrac{5}{2}\,\mathrm{I}\sqrt{2},\,2-\tfrac{5}{2}\,\mathrm{I}\sqrt{2}\Big],\,\Big[2-5\,\mathrm{I}\sqrt{2}\Big],\,\tfrac{199}{200}\Big)\sqrt{2}\\
& +30646\,\mathrm{hypergeom}\Big(\Big[2+\tfrac{5}{2}\,\mathrm{I}\sqrt{2},\,2+\tfrac{5}{2}\,\mathrm{I}\sqrt{2}\Big],\,\Big[2+5\,\mathrm{I}\sqrt{2}\Big],\,\tfrac{199}{200}\Big)\,\mathrm{hypergeom}\Big(\Big[1-\tfrac{5}{2}\,\mathrm{I}\sqrt{2},\,1-\tfrac{5}{2}\,\mathrm{I}\sqrt{2}\Big],\,\Big[1\\
& -5\,\mathrm{I}\sqrt{2}\Big],\,\tfrac{199}{200}\Big)+49750\,\mathrm{I}\,\mathrm{hypergeom}\Big(\Big[2+\tfrac{5}{2}\,\mathrm{I}\sqrt{2},\,2+\tfrac{5}{2}\,\mathrm{I}\sqrt{2}\Big],\,\Big[2+5\,\mathrm{I}\sqrt{2}\Big],\,\tfrac{199}{200}\Big)\sqrt{2}\,\mathrm{hypergeom}\Big(\Big[1\\
& -\tfrac{5}{2}\,\mathrm{I}\sqrt{2},\,1-\tfrac{5}{2}\,\mathrm{I}\sqrt{2}\Big],\,\Big[1-5\,\mathrm{I}\sqrt{2}\Big],\,\tfrac{199}{200}\Big)\Big)-\Big(10\,\Big(102000\,\mathrm{I}\,\mathrm{hypergeom}\Big(\Big[1-\tfrac{5}{2}\,\mathrm{I}\sqrt{2},\,1-\tfrac{5}{2}\,\mathrm{I}\sqrt{2}\Big],\\
& \Big[1-5\,\mathrm{I}\sqrt{2}\Big],\,\tfrac{199}{200}\Big)\sqrt{2}+8119200\,\mathrm{hypergeom}\Big(\Big[1-\tfrac{5}{2}\,\mathrm{I}\sqrt{2},\,1-\tfrac{5}{2}\,\mathrm{I}\sqrt{2}\Big],\,\Big[1-5\,\mathrm{I}\sqrt{2}\Big],\,\tfrac{199}{200}\Big)
\end{aligned}
$$

$$
\begin{aligned}
& -30646\,\mathrm{hypergeom}\Big(\Big[2-\tfrac{5}{2}\,\mathrm{I}\sqrt{2},\,2-\tfrac{5}{2}\,\mathrm{I}\sqrt{2}\Big],\,\Big[2-5\,\mathrm{I}\sqrt{2}\Big],\,\tfrac{199}{200}\Big)+49750\,\mathrm{I}\,\mathrm{hypergeom}\Big(\Big[2-\tfrac{5}{2}\,\mathrm{I}\sqrt{2},\,2\\
& -\tfrac{5}{2}\,\mathrm{I}\sqrt{2}\Big],\,\Big[2-5\,\mathrm{I}\sqrt{2}\Big],\,\tfrac{199}{200}\Big)\sqrt{2}\Big)\Big(-200\,\mathrm{e}^{-\frac{1}{2}\mathrm{I}\sqrt{2}\,x}+199\,\mathrm{e}^{-\frac{1}{2}\mathrm{I}\sqrt{2}\,x-\frac{1}{5}x}\Big)\,\mathrm{hypergeom}\Big(\Big[1+\tfrac{5}{2}\,\mathrm{I}\sqrt{2},\,1\\
& +\tfrac{5}{2}\,\mathrm{I}\sqrt{2}\Big],\,\Big[1+5\,\mathrm{I}\sqrt{2}\Big],\,\tfrac{199}{200}\,\mathrm{e}^{-\frac{1}{5}x}\Big)\Big)\Big/\Big(204000\,\mathrm{I}\,\mathrm{hypergeom}\Big(\Big[1+\tfrac{5}{2}\,\mathrm{I}\sqrt{2},\,1+\tfrac{5}{2}\,\mathrm{I}\sqrt{2}\Big],\,\Big[1+5\,\mathrm{I}\sqrt{2}\Big],\\
& \tfrac{199}{200}\Big)\sqrt{2}\,\mathrm{hypergeom}\Big(\Big[1-\tfrac{5}{2}\,\mathrm{I}\sqrt{2},\,1-\tfrac{5}{2}\,\mathrm{I}\sqrt{2}\Big],\,\Big[1-5\,\mathrm{I}\sqrt{2}\Big],\,\tfrac{199}{200}\Big)-30646\,\mathrm{hypergeom}\Big(\Big[1+\tfrac{5}{2}\,\mathrm{I}\sqrt{2},\,1\\
& +\tfrac{5}{2}\,\mathrm{I}\sqrt{2}\Big],\,\Big[1+5\,\mathrm{I}\sqrt{2}\Big],\,\tfrac{199}{200}\Big)\,\mathrm{hypergeom}\Big(\Big[2-\tfrac{5}{2}\,\mathrm{I}\sqrt{2},\,2-\tfrac{5}{2}\,\mathrm{I}\sqrt{2}\Big],\,\Big[2-5\,\mathrm{I}\sqrt{2}\Big],\,\tfrac{199}{200}\Big)\\
& +49750\,\mathrm{I}\,\mathrm{hypergeom}\Big(\Big[1+\tfrac{5}{2}\,\mathrm{I}\sqrt{2},\,1+\tfrac{5}{2}\,\mathrm{I}\sqrt{2}\Big],\,\Big[1+5\,\mathrm{I}\sqrt{2}\Big],\,\tfrac{199}{200}\Big)\,\mathrm{hypergeom}\Big(\Big[2-\tfrac{5}{2}\,\mathrm{I}\sqrt{2},\,2-\tfrac{5}{2}\,\mathrm{I}\sqrt{2}\Big],\\
& \Big[2-5\,\mathrm{I}\sqrt{2}\Big],\,\tfrac{199}{200}\Big)\sqrt{2}+30646\,\mathrm{hypergeom}\Big(\Big[2+\tfrac{5}{2}\,\mathrm{I}\sqrt{2},\,2+\tfrac{5}{2}\,\mathrm{I}\sqrt{2}\Big],\,\Big[2+5\,\mathrm{I}\sqrt{2}\Big],\,\tfrac{199}{200}\Big)\,\mathrm{hypergeom}\Big(\Big[1\\
& -\tfrac{5}{2}\,\mathrm{I}\sqrt{2},\,1-\tfrac{5}{2}\,\mathrm{I}\sqrt{2}\Big],\,\Big[1-5\,\mathrm{I}\sqrt{2}\Big],\,\tfrac{199}{200}\Big)+49750\,\mathrm{I}\,\mathrm{hypergeom}\Big(\Big[2+\tfrac{5}{2}\,\mathrm{I}\sqrt{2},\,2+\tfrac{5}{2}\,\mathrm{I}\sqrt{2}\Big],\,\Big[2+5\,\mathrm{I}\sqrt{2}\Big],\\
& \tfrac{199}{200}\Big)\sqrt{2}\,\mathrm{hypergeom}\Big(\Big[1-\tfrac{5}{2}\,\mathrm{I}\sqrt{2},\,1-\tfrac{5}{2}\,\mathrm{I}\sqrt{2}\Big],\,\Big[1-5\,\mathrm{I}\sqrt{2}\Big],\,\tfrac{199}{200}\Big)\Big)
\end{aligned}
$$

plot(*funcy*(*x*), *x* = 0 ..50);

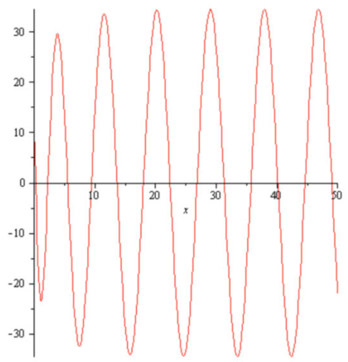


Fig. S 8b

Supplementary 9: Solutions with Learning Effects

Supplementary 9a

; $dgl := diff(y(x), x, x) + \exp(x) \cdot y(x) = 0;$

$$\frac{d^2}{dx^2} y(x) + + e^x y(x) = 0$$

$dsolve(\{dgl, y(0) = 0, D(y)(0) = 1\}, y(x));$

$$y(x) = \frac{\mathrm{BesselY}(0,2)\,\mathrm{BesselJ}\left(0, 2\,e^{\frac{1}{2}x}\right)}{-\mathrm{BesselY}(0,2)\,\mathrm{BesselJ}(1,2) + \mathrm{BesselY}(1,2)\,\mathrm{BesselJ}(0,2)} - \frac{\mathrm{BesselJ}(0,2)\,\mathrm{BesselY}\left(0, 2\,e^{\frac{1}{2}x}\right)}{-\mathrm{BesselY}(0,2)\,\mathrm{BesselJ}(1,2) + \mathrm{BesselY}(1,2)\,\mathrm{BesselJ}(0,2)}$$

$funcy := unapply(rhs(dsolve(\{dgl, y(0) = 0, D(y)(0) = 1\}, y(x))), x);$

$$x \rightarrow \frac{\mathrm{BesselY}(0,2)\,\mathrm{BesselJ}\left(0, 2\,e^{\frac{1}{2}x}\right)}{-\mathrm{BesselY}(0,2)\,\mathrm{BesselJ}(1,2) + \mathrm{BesselY}(1,2)\,\mathrm{BesselJ}(0,2)} - \frac{\mathrm{BesselJ}(0,2)\,\mathrm{BesselY}\left(0, 2\,e^{\frac{1}{2}x}\right)}{-\mathrm{BesselY}(0,2)\,\mathrm{BesselJ}(1,2) + \mathrm{BesselY}(1,2)\,\mathrm{BesselJ}(0,2)}$$

$plot(funcy(x), x = -2..5);$

$plot(funcy(x), x = -1..100);$

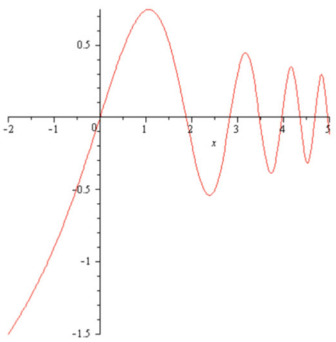


Fig. S 9a

Supplementary 9b

$dgl := diff(y(x), x, x) + (1 + x) \cdot y(x) = 0;$

$$\frac{d^2}{dx^2} y(x) + (1 + x)\, y(x) = 0$$

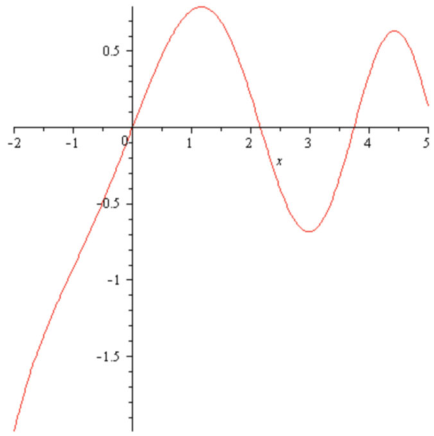
$dsolve(\{dgl, y(0) = 0, D(y)(0) = 1\}, y(x));$

$$y(x) = \frac{\mathrm{AiryBi}(-1)\, \mathrm{AiryAi}(-1 - x)}{-\mathrm{AiryBi}(-1)\, \mathrm{AiryAi}(1, -1) + \mathrm{AiryBi}(1, -1)\, \mathrm{AiryAi}(-1)} - \frac{\mathrm{AiryAi}(-1)\, \mathrm{AiryBi}(-1 - x)}{-\mathrm{AiryBi}(-1)\, \mathrm{AiryAi}(1, -1) + \mathrm{AiryBi}(1, -1)\, \mathrm{AiryAi}(-1)}$$

$funcy := unapply(rhs(dsolve(\{dgl, y(0) = 0, D(y)(0) = 1\}, y(x))), x);$

$$x \rightarrow \frac{\mathrm{AiryBi}(-1)\, \mathrm{AiryAi}(-1 - x)}{-\mathrm{AiryBi}(-1)\, \mathrm{AiryAi}(1, -1) + \mathrm{AiryBi}(1, -1)\, \mathrm{AiryAi}(-1)} - \frac{\mathrm{AiryAi}(-1)\, \mathrm{AiryBi}(-1 - x)}{-\mathrm{AiryBi}(-1)\, \mathrm{AiryAi}(1, -1) + \mathrm{AiryBi}(1, -1)\, \mathrm{AiryAi}(-1)}$$

$plot(funcy(x), x = -2\,..5);$



$plot(funcy(x), x = -1\,..100);$

Fig. S 9b

Supplementary 10

The combination of a spontaneous with an accumulative reaction leads – in addition to the C-path – to a damped oscillatory circuit. The sociological equivalent circuit shown in Fig. Suppl. 10 is isomorphic to an electrical parallel LCR circuit.

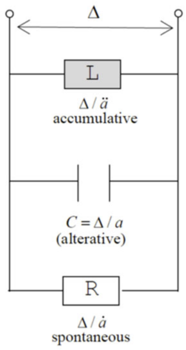


Fig. S10

While an LC circuit leads to harmonic oscillations and an RC circuit to exponential relaxations, the LCR circuit shows exponentially damped oscillations, the frequency of which is constant and determined by L, C, R (see textbooks of physics). The terms “accumulative”, “alterative” and “spontaneous” refer to the response of the rate $\dot{a}$ on the driving function Δ and thus deviates from the convention in electrodynamics. The parallel switching of R can be connected with the opening of a new mechanistic channel (implying a partitioning of the rate $\dot{a}$). The effect of a serial implementation of R into a LC circuit would be a decrease of the reaction rate $\dot{a}$.

In the case of a mechanical oscillation, L would correspond to mass m characterizing the inertia, and C to $1/D$ where D is the spring constant. $1/D$ corresponds to a parameter characterizing the mechanical yielding (C: dielectric yielding) or generally to a permittance. R would correspond to mechanical resistance (friction). The mechanical analogue may even offer a better comparison than the electromagnetic analogue.

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